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(De)construct for Circular Economy

(Des)construir para a Economia Circular

WP 5 - Model

5.1 - Mathematical model for the location of Recovery and Sorting centres

5.2 - Mathematical model for the inter-municipal network for the reuse of materials

Final report

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Content

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Table of Figures

List of Tables

List of acronyms used

CDW - Construction and demolition wastes

mCDW - Mixed construction and demolition wastes

RSS - Recovery Sorting and Storing

1. Introduction

The main objective of the project **(De)construct for Circular Economy [in Portuguese: (Des)construir para a Economia Circular]** is to promote a regional strategy for the reuse of building products and components as well as the recycling of construction and demolition waste (CDW), thus reducing the environmental impact of the construction sector and promoting its circularity.

Task WP5 – Development of mathematical models:

Given the data scarcity, this report presents a mathematical model that integrates both mathematical models from:

- Activity 5.1 –Mathematical model for the location of Recovery and Sorting centres
- Activity 5.2 – Mathematical model for the inter-municipal network for the reuse of materials

The merging of the two models is a direct result of an effort to allow better data coupling between the models, to avoid data repetition and linkage errors, and to facilitate the configuration of the models' case study and scenarios by less experient users.

The report is structured in the next chapters:

- Introduction (present chapter);
- Objectives (chapter 2)
- Methodology (chapter 3);
- About the recovery, sorting, storing, and refitting network. (chapter 4);
- Case study definition (chapter 5);
- Parameters of the Base Scenario (chapter 6);
- General definitions for scenario building (chapter 7);
- Scenarios definition and optimized results (chapter 8);
- Results comparison (chapter 9);
- Deliverables (chapter 10);
- Conclusions (chapter 11);
- Sets and Parameters of the Case Study (Annex I)
- Wide list of optimized scenarios' results (Annex II)
- Mathematical Model (Annex III)

(This is the final version of the report.)

2. Objectives

The objective of this task is to develop mathematical models to help the strategic, tactical and operational decisions for the implementation of an inter-municipal recovery, sorting, and storing network of CDWs and reusable materials locally produced by deconstruction (selective demolition) processes, to increase the circular economy while accounting for the social side by including local social organizations. Depending on the conditions imposed, the model gives the optimal network design defining the optimal location to install the required processing centres, and their respective capacities, and shows the optimized material flow in the network.

3. Methodological approach

The development of the models uses a representation where the materials are univocally represented by states that flow in the network to be processed by the task performed by the network processing units, This representation is usually known in the specialized literature as a state task network, STN for short.

To ensure the model's generality, its implementation used a model core, constituted by the mathematical equations, and a parametrization warranted by external files. One of these files defines the model sets and parameters that remain immutable, and a second one defines the parameters for the particular scenario to be optimized.

To verify the model’s suitability, executability, and general behaviour, an inter-municipal case study was used involving all the municipalities involved in the present project. The possible types of the network’s centres/units are *a)* the recovery, sorting, and storing (RSS) centres, *b)* the crushing units, and *c)* the refitting centres for processing the reusable materials (RM).

Following a request by IrRADIARE two sets of CDW production were used, one obtained from APA and the second one from IrRADIARE that adds the illegal deposition values. The model locations are defined at a municipal level with their main city as the geographic point for defining the transport routes and the possible installation locations. A set of scenarios was defined from the type of CDW data used, and the values used for the RM considered (only doors and windows were used).

The model allows us to define the existence of pre-installed centres/units, to limit the number of any given type of centre/unit, their maximal capacity and the interconnections allowed, among others.

To avoid transport costs the crushing centres are defined to be located at an RSS centre. However, if a particularly large demand for crushed material occurs in any other network location, it suffices to consider a fictitious RSS centre with the crushing unit at that location. This is made possible because the RSS operative costs for all the crushable materials are simply the costs of loading that material into the crushing process or storage.

All the optimal install locations along with other pertinent results for any given scenario defined are hereby shown in the individual sub-chapter specific to that scenario definition and results.

4. About the recovery, sorting, storing, and refitting network.

As part of task 5.1 of the (De)Construct project, a mathematical model was developed for a network for the Recovery, Sorting, and Storage (RSS) of CDWs and Reusable Materials (RMs), which includes the task 5.2 network by including a crushing unit and the RMs refitting unit.

The so-called reusable materials are products from previous processes of deconstruction or demolition of buildings, not included in this model. Figure 1 shows a simplified schematic of material flows in the RSS network.

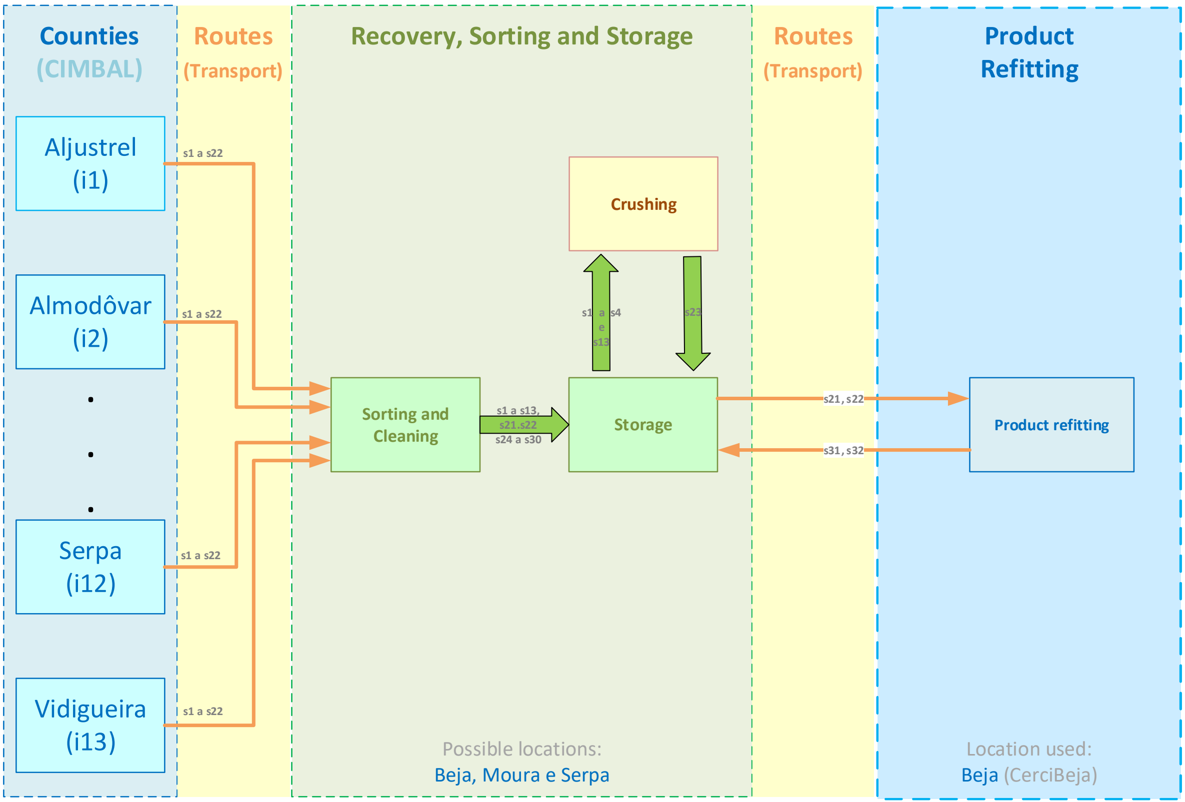

Figure 1 - Flow diagram of CDWs and RMs in an RSS network, with crushing and refitting units.

The mathematical model was developed to be as general as possible; either from the point of view of the geographic scale, the entities involved, or even the characteristics of the respective transports and transformation processes.

To achieve this generality, the core of the model, *i.e.*, the set of mathematical equations, was separated from its parameterization. The latter is achieved by including the file(s) that define the values of the parameter and constants needed to optimize the model. Thus, it is possible to change the case study and all its respective scenarios if the general layout of the material flow shown in Figure 1 is followed.

The independent parameterization allows the user to define, among others, the geographic scale to be used by specifying the locations, for example, at the level of the parish or the metropolitan region, and the possible locations for the installation of units/centres among them, while specifying whether the crushing and recovery of RMs are installed, and all the recipes of the processes involved, etc…

The RSS centres store all the materials and products on the network and can house crushing units on their premises, being then the only receiver of the respective products.

The core of the model needs a "dummy" process, *k0*, to be installed in each one of the locations considered to load the CDWs and RMs and send them to the RRS centres, thus allowing the assessment of the respective transport costs and guaranteeing that all inputs are processed.

The mathematical modelling is based on the quantities of the network's materials flow between the considered processes which are schematically represented in Figure 2 for some typical input materials.

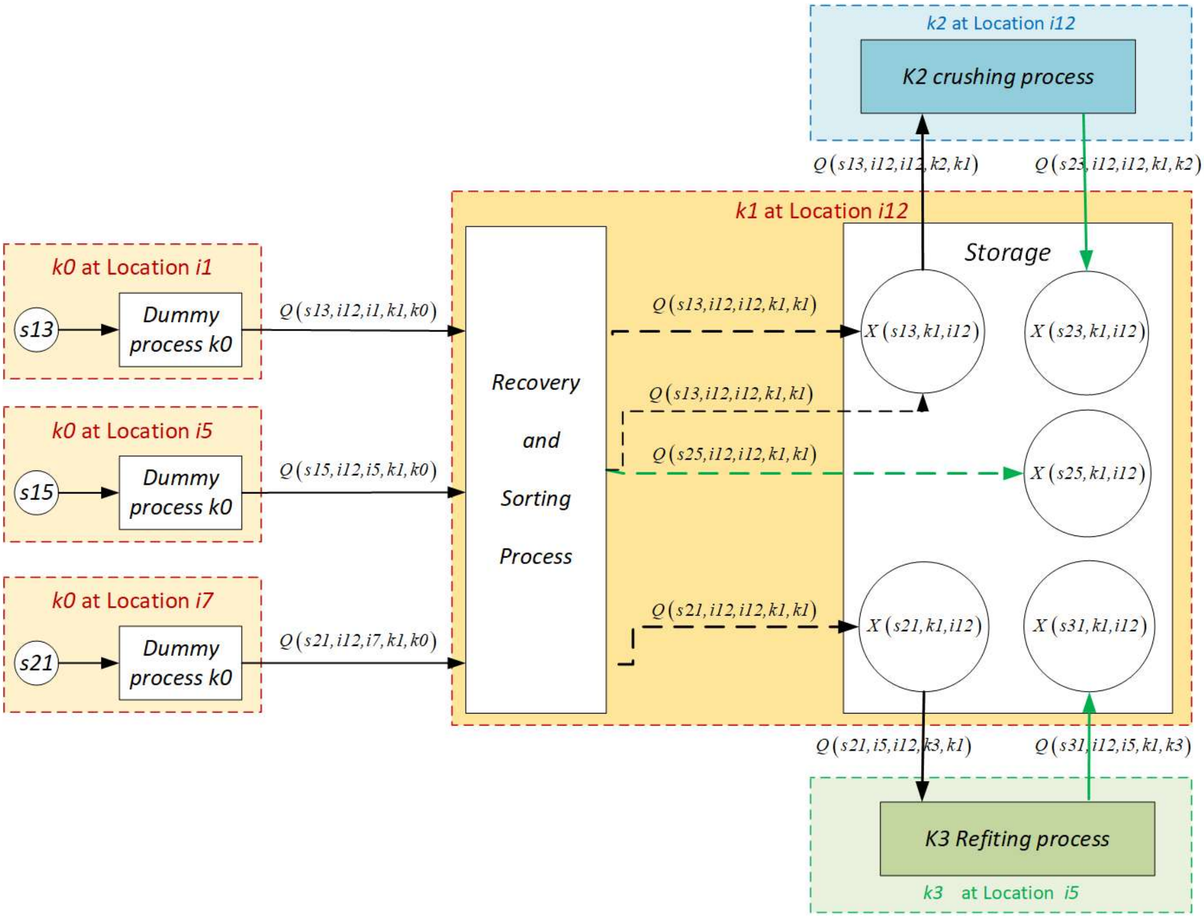

Figure 2 - The network flow for some representative input materials.

The positive variable $Q(s,i,j,k,kp)$ is a mathematical representation of the quantity of product *s* going from a process *kp* located at *j* to be processed by *k* at location *i*.

For a detailed description of the complete mathematical model see annex III.

5. Case study definition

The case study used to validate the model considers the area of Baixo Alentejo, which covers 13 counties (*municípios*) in the district of Beja delineated in Figure 3 by the highlighted red area on the map of Portugal.

Figure 3 - Case study region, 13 counties in Baixo Alentejo.

The case study materials are the inputs, a collection of CDWs and RMs presented in Table 1along with all their possible processed products. The CDW types are the ones identified by the EU regulations while the RMs were either suggested at project meetings or have foreseeable value after refitting. However the type of materials accounted for in the case study are the mixed construction and demolition wastes (mCDW), and the reusable materials (RM) used are doors and windows from selective demolition processes.

The case study assumes that the locations producing CDWs and RMs are at a county level and that the routes are defined using the county head cities as endpoints. Table 2 presents the name of these head cities which are used by the case study as the provenance location for the input materials.

The identification of all possible processing units/centres is presented in Table 3.

The eventual locations to install all units/centres were suggested by ResiAlentejo based on historical CDW production and are presented in Table 4.

The distances of all possible routes presented in Table 5 were obtained from Google© maps using the option no highways or boots.

Important note: All the remaining case study parameters are gathered from an Excel data file sent to the pertinent project partners for analysis and approval, and constitute the possible ones to gather from the published and online data.

The main elements defining the case study are identified and presented next, for a complete description of the case study parameter values see *Annex I*.

Table 1 - Set s identifying all states

Element	Name	
s1	Concrete	CDWs Materials
s2	Bricks	CDWs Materials
s3	Tiles and Ceramics	CDWs Materials
s4	Concrete bricks tiles and ceramics Mix	CDWs Materials
s5	Wood	CDWs Materials
s6	Glass	CDWs Materials
s7	Plastic	CDWs Materials
s8	Bituminous mixtures	CDWs Materials
s9	Mixed metals	CDWs Materials
s10	Insulation materials	CDWs Materials
s11	Gypsum-based construction materials	CDWs Materials
s12	CDW containing dangerous substances	CDWs Materials
s13	Mix construction and demolition wastes	CDWs Materials
s14	Reusable roof Tiles	Reusable Materials
s15	Reusable Decorative wall tiles	Reusable Materials
s16	Reusable Decorative floor tiles	Reusable Materials
s17	Reusable Decorative Marble	Reusable Materials
s18	Reusable Decorative Stone	Reusable Materials
s19	Reusable Masonry Stone	Reusable Materials
s20	Reusable Structural wood	Reusable Materials
s21	Reusable Doors	Reusable Materials
s22	Reusable windows	Reusable Materials
s23	Filling Material	Products
s24	Processed Roof Tiles	Products
s25	Processed Decorative wall tiles	Products
s26	Processed Decorative floor tiles	Products
s27	Processed Decorative Marble	Products
s28	Processed Decorative Stone	Products
s29	Processed Masonry Stone	Products
s30	Processed Structural wood	Products
s31	Processed Doors	Products
s32	Processed windows	Products

Table 2 - Set i of county's names

Element	Name
i1	Aljustrel
i2	Almodôvar
i3	Alvito
i4	Barrancos
i5	Beja
i6	Castro Verde
i7	Cuba
i8	Ferreira do Alentejo
i9	Mértola
i10	Moura
i11	Ourique
i12	Serpa
i13	Vidigueira

Table 3 - Set k of processing units

Element	Process Name
k0	Dummy process
k1	Recover, sorting, storing
k2	Crushing and sieving
k3	Repair and fitting

Table 4 - Set r of possible install locations

Element	Name
r5	Beja
r10	Moura
r12	Serpa

Table 5 - Distances from r(i) to i [km]

	i1	i2	i3	i4	i5	i6	i7	i8	i9	i10	i11	i12	i13
i5	38.3	71.1	37.0	101.0	5.0	23.2	20.0	24.0	53.0	52.1	73.1	32.9	25.1
i10	87.0	118.0	61.7	49.8	52.3	95.1	45.7	69.2	88.5	5.0	109.0	34.0	35.8
i12	65.4	86.2	62.4	76.6	28.0	63.2	46.4	50.6	57.1	34.9	77.5	5.0	43.7

To allow the set elements to appear more than once in a variable or equation, a set of aliases is defined, $\{i\}=\{j\}$, $\{k\}=\{kp\}$, $\{s\}=\{sp\}$, with the elements in the same order.

6. Parameters of the Base Scenario

The data presented next are the values used to define a base scenario. Most of the values presented here are common to all the defined scenarios. These are part of the values that were sent to project partners for analysis and approval, and constitute the possible ones to gather from published and online data.

All the modifications imposed by the scenario's configuration are reported in the scenario description.

Table 6 - General purpose scalars

Scalar	Text description	Value
Load	Individual transport max load [t]	5
LoadCost	Transport Loading Cost [€ /t]	2
Ndoors	Number of doors per tonne	40
NWindows	Number of windows per tonne	50
DWx	Overcharge factor for Windows and Doors transport	5

Parameter values

Table 7 - Dem(s), Quantity demanded for material at state s [t]

s23	*s31*	*s32*
13869	1	0.96

Table 8 - Nk(k), Admissible number of process centres

k0	*k1*	*k2*	*k3*
13	2	2	1

Table 9 - State s value VALUEs(s) and cost (Voucher) [€]

$Voucher(s)$ Cost		$Vals(s)$ Value		
s21	*s22*	*s23*	*s31*	*s32*
3600	3750	20	12000	12500

7. General definitions for scenario building

Two root scenarios are defined, a scenario Type A - using the RCDs data for the region under study for 2020, kindly made available by the Environmental Portuguese Agency (APA), and a root scenario of type B that adds the illegally deposed CDW values to the values from the previous scenario. The use of the two types of root scenarios allows for verifying if the variation in the recoverable materials' quantities leads to noticeable variations in the results.

The scenarios are built essentially by changing the demand values for the crushed material and the refitted doors and windows. The use of reusable materials demand serves essentially to test the model.

Table ***10*** presents the recoverable material quantities used for both types of scenarios, while Table 11 presents the eventual demand values used for the crushed and refitted products.

Table 10 - CDWs and Reusable Materials considered for scenarios of the type A and B.

Location		Scenario of Type A			Scenario of Type B		
Name	ID	*s13*	*s21*	*s22*	*s13*	*s21*	*s22*
Aljustrel	i1	145.782			350.000		
Almodôvar	i2	1900.197			1910.000		
Alvito	i3	120.840			1323.000		
Barrancos	i4				186.000		
Beja	i5	1682.241	0.325	0.320	1810.000	0.325	0.320
Castro Verde	i6	2061.120			2221.000		
Cuba	i7						
Ferreira do Alentejo	i8	898.150			1020.000		
Mértola	i9	239.430			1526.000		
Moura	i10	242.200	0.325	0.330	294.000	0.325	0.330
Ourique	i11	88.040			429.000		
Serpa	i12		0.350	0.320	750.000	0.350	0.320
Vidigueira	i13	32.120			44.000		

Table 10 of any of the s14 to s20 materials, to be cleaned and sorted by a k1 unit, is due to the lack of a proper characterization of the respective process's recipes, the quantities available and their behaviour under optimization is similar to that of other materials tested.

Table 11 - Crushed material and refitted products demand used

Name	Product		
	Crushed materials	***Doors***	***Windows***
ID	***s23***	***s31***	***s32***
Quantity (t)	7400	1	0.95
#		40	38

All scenarios are solved using GAMS 34.1.0 running on a Windows 10 Pro desktop equipped with an AMD Ryzen 7 3700X 8-Core Processor, 3.70 GHz with 64GB RAM.

Annex II shows the general scenario that is the base for all the specific ones. A brief description of the individual scenarios is presented next, for a more complete description see Annex III.

8. Scenarios definition and optimized results

Scenario A1

This scenario A1 uses the quantities of recoverable materials presented for a type A scenario and the demand values of *s23*=0, s*31*=0, and *s32*= 0 [t].

This scenario optimization uses 9732 variables, and the computational resources of 0.11 s to generate and 0.078 s to execute, leading to a minimized total cost of 98715€.

The optimal value is obtained by installing a *k1* unit in an RSS centre at Beja with the null demand imposition leading to no additional processing unit being installed to avoid increasing the network's total costs. The referred cost reduction occurs by avoiding the transport costs of the reusable and the refitted materials, respectively, to and from the pre-existing *k3* unit.

The choice of Beja to install the RSS centre is a direct consequence of the minimization of the network transport costs given that all the investment and the operational components of the cost are the same for all *k1*, independently of their location.

The RSS assessed network costs are 77622€ for investment, 15683€ for transportation, and 5410€ for processes' operation leading to a total cost of 98715€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS centre at Beja stores 7409 metric tonnes of mCDW, 40 doors, and 49 windows, respectively the equivalent of 1 tonne of doors and 0.98 tonnes of windows to be refitted.

This scenario receives only the materials recovered from all counties, with no *k1* sorting or cleaning because the type of materials involved demands no further processing by *k1*, no crushing by *k2*, nor refitting by the existent *k3* to avoid increasing the costs.

Scenario A2

This scenario uses the quantities of the recoverable material presented for scenario type A and the demand values of *s23*=7400, *s31*=0, and *s32*= 0 [t].

The optimization of this scenario uses 9732 variables, and the computational resources of 0.14 s to generate and 0.047 s to execute, leading to a minimized total cost of 118991 €

The optimal value is obtained by installing a *k1* and a *k2* unit at an RSS centre at Beja with the null demand values for refitted doors and windows imposing that the existing refitting unit is not used to avoid the unnecessary additional network costs – thus eliminating the need for transporting materials to and from *k3* and to process the corresponding reusable materials.

The RSS assessed network costs are 86872€ for investment, 20123€ for transportation, and 11996€ for processes' operation leading to a total cost of 98715€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS network total value of 148000€ is assessed for the crushed material produced with a market value of 20€/t.

The RSS centre at Beja stores 9 metric tonnes of mCDW, 7400 tonnes of crushed material, 40 doors, and 49 windows.

Scenario A3

This scenario uses the recoverable material values presented for scenario type A and the demand values of *s23*=7400, *s31*=1, and *s32*= 0.95 [t].

This scenario optimization uses 9732 variables, and the computational resources of 0.125 s to generate and 0.063 s to execute, leading to a minimized total cost of 127437 €

The optimal value is obtained by installing a *k1* and a *k2* unit in an RSS centre at Beja, the location *i5*, and by using the available *k3* pre-installed unit at Beja.

The RSS assessed network costs are 86872€ for investment, 20123€ for transportation, and 20436€ for processes' operation leading to a total cost of 127437€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS centre stores 9 tonnes of mCDW, 1 recovered window, 7400 tonnes of crushed material, 40 refitted doors, and 48 refitted windows.

The RSS network value of 148000€ of crushed material, 12000€ of doors, and 12000€ of windows, for a total value of 172000€.

Scenario B1

This scenario uses the recoverable material values presented for scenario type B and the demand values of s23=0, s31=0, and s32= 0 [t].

This scenario optimization uses 9732 variables, and the computational resources of 0.156 s to generate and 0.109 s to execute, leading to a minimized total cost of 112972€

The optimal value is obtained by installing a *k1* unit in an RSS centre at Beja and not using the available *k3* pre-installed unit at Beja. The RSS network's costs are shown in table II.14.

The RSS assessed network costs are 77622€ for investment, 26689€ for transportation, and 8661€ for processes' operation leading to a total cost of 112972€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS centre stores 9 tonnes of mCDW, 40 recovered doors, and 49 recovered windows.

Scenario B2

This scenario uses the recoverable material values in a type B scenario and the demand values of s23=7400, s31=0, and s32= 0 [t].

The optimization of this scenario uses 9732 variables, and the computational resources of 0.094 s to generate and 0.063 s to execute, leading to a minimized total cost of 133248 €

The optimal value is obtained by installing a *k1* and a *k2* unit at an RSS centre at Beja with the null demand values for refitted doors and windows imposing that the existing refitting unit is not used to avoid the unnecessary additional network costs.

The RSS assessed network costs are 86872€ for investment, 31129€ for transportation, and 15247€ for processes' operation leading to a total cost of 133248€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS network total value of 148000€ is assessed for the crushed material produced with a market value of 20€/t.

The RSS centre at Beja now recovers 11863 tonnes of mCDW, stores 4463 tonnes of mCDW, 7400 tonnes of crushed material, 40 recovered doors, and 49 recovered windows.

Scenario B3

This scenario uses the recoverable material values presented for scenario type B, and the demand values of s23=11863, s31=0, and s32= 0 [t].

This scenario optimization uses 9734 variables, and the computational resources of 0.094 s to generate and 0.078 s to execute, leading to a minimized total cost of 139898 €

Despite the increase in the *s23* demand, the optimal value is again obtained by installing a *k1* and a *k2* unit in an RSS centre at Beja, the location *i5*, and by not using the available *k3* pre-installed unit at Beja. The RSS network's costs are shown in table II.19.

The RSS assessed network costs are 86872€ for investment, 33806€ for transportation, and 19220€ for processes' operation leading to a total cost of 139898€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS network total value of 237260€ is assessed for the crushed material produced with a market value of 20€/t.

The RSS centre recovers 11863 tonnes of mCDW, and stores 40 recovered doors, 49 recovered windows, and 11863 tonnes of crushed material.

The RSS network value of 237260€ of crushed material for a total value of 237260€.

Scenario B4

This scenario uses the recoverable material values presented for scenario type B with a 5000 tonne of *s13* now available at location *i10*, and the demand values of *s23*=11863, *s31*=0, and *s32*= 0 [t]. The main purpose of this scenario is to test for a change in *k1* install location.

This scenario optimization uses 9732 variables, and the computational resources of 0.125s to generate and 0.063s to execute, leading to a minimized total cost of 158518€

Given the increase in the *s13* material available at Moura and the subsequent demand of s23, the optimal value is now obtained by installing a *k1* and a *k2* unit in an RSS centre at Moura, the model's location *i10*, and by not using the available *k3* pre-installed unit at Beja.

The RSS assessed network costs are 86872€ for investment, 48991€ for transportation, and 22655€ for processes' operation leading to a total cost of 158518€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS network total value of 237260€ is assessed for the crushed material produced.

The RSS centre at Beja now recovers 16569 tonnes of mCDW, stores 4706 tonnes of mCDW, 11863 tonnes of crushed material, 40 recovered **doors, and 49** recovered **windows**.

Scenario B5

This scenario uses the recoverable material values of scenario type B, and the demand values of *s23*=7400, *s31*=1, and *s32*= 0.96.

This scenario optimization uses 9732 variables, and the computational resources of 0.109 s to generate and 0.078 s to execute, leading to a minimized total cost of 141694 €

B5 scenario optimal value is now obtained by installing a *k1* and a *k2* unit in an RSS centre at Beja, the location *i5*, and by using the available *k3* pre-installed unit at Beja.

The RSS assessed network costs are 86872€ for investment, 31134€ for transportation, and 23688€ for processes' operation leading to a total cost of 141694€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS centre recovers 11863 tonnes of mCDW, stores 9 tonnes of mCDW, 1 recovered window, 7400 tonnes of crushed material, 40 refitted doors, and 48 refitted windows.

The RSS network value of 148000€ of crushed material, 12000€ of doors, and 12000€ of windows, for a total value of 172000€.

Scenario B6

This scenario uses the recoverable material values of scenario type B, and the demand values of *s23*=11863, *s31*=1, and *s32*= 0.96.

This scenario optimization uses 9732 variables, and the computational resources of 0.109 s to generate and 0.125 s to execute, leading to a minimized total cost of 148344 €

B6 scenario optimal value is now obtained by installing a *k1* and a *k2* unit in an RSS centre at Beja, the location *i5*, and by using the available *k3* pre-installed unit at Beja.

The RSS assessed network costs are 86872€ for investment, 33812€ for transportation, and 27660€ for processes' operation leading to a total cost of 148344€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS network value of 148000€ of crushed material, 12000€ of doors, and 12000€ of windows, for a total value of 172000€.

The RSS centre recovers 11863 tonnes of mCDW, stores 1 recovered window, 11863 tonnes of crushed material, 40 refitted doors, and 48 refitted windows.

Scenario B7

This scenario uses the recoverable material values of scenario type B with a 2300 tonne of s13 now available at location i10, and the demand values of *s23*=11863, *s31*=1, and *s32*= 0.96.

This scenario optimization uses 9732 variables, and the computational resources of 0.125 s to generate and 0.156 s to execute, leading to a minimized total cost of 161828 €

B7 scenario optimal value is now obtained by installing a k1 and a k2 unit in an RSS centre at Moura, location i10, and by using the available k3 pre-installed unit at Beja.

The RSS assessed network costs are 86872€ for investment, 44046€ for transportation, and 30909€ for processes' operation leading to a total cost of 161828€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS network value of 277380€ of crushed material, 12000€ of doors, and 12000€ of windows, for a total value of 301380€.

The RSS centre recovers 13869 tonnes of mCDW, stores 1 recovered window, 11863 tonnes of crushed material, 40 refitted doors, and 48 refitted windows.

This scenario proves that it is enough to "modestly" raise the mCDW quantity, for example from 294t to 2300t of mCDW at Moura, in the vicinity of a possible location to install a k1 RSS centre to get *k1* not installed at Beja. It happens when the costs of transporting such a quantity to and from that location, to all other locations and units including Beja's *k3,* compensate for the eventual transport costs to Beja's k1 RSS centre including to its collocated k3 refitting centre.

Scenario B8

This scenario uses the recoverable material values of scenario type B and upper limits the k1 capacity to 8000 tonnes, and the demand values of *s23= 13869, s31=1, and s32= 0.96.*

This scenario optimization uses 9732 variables, and the computational resources of 0.094 s to generate and 0.094 s to execute, leading to a minimized total cost of 243451€

B8 scenario optimal value is now obtained by installing a *k1* and a *k2* unit in RSS centres at Beja and Moura, and by using the available *k3* pre-installed unit at Beja. Bejas RSS centre uses the maximum allowed capacity (8000 tonnes) while Moura installs 5870.98.

The RSS assessed network costs are 173744€ for investment, 38797€ for transportation, and 30909€ for processes' operation leading to a total cost of 243451€. It may also be accounted for an additional cost of 7275€ for RM vouchers used to stimulate the reuse of materials.

The RSS network value of 277380€ of crushed material, 12000€ of doors, and 12000€ of windows, for a total value of 301380€.

The RSS centre recovers 13869 tonnes of mCDW, stores 1 recovered window, 11863 tonnes of crushed material, 40 refitted doors, and 48 refitted windows.

The opening of two RSS centres doubles the installation costs but it diminishes the transportation costs. Moura stores a recovered window from Moura that is not needed to fulfil the imposed demand because it avoids transportation costs and Beja RSS is already at full capacity.

This scenario proves that if the allowed maximal capacity for k1 RSS centres is insufficient for the quantity of the available recoverable material, two or more RSS centres are then open. It needs to be emphasized here, that the total number of possible centres of type *k* to be installed is limited by the *Nk(k)* parameter value which when insufficient for the needs forces the optimization to fail.

9. Results comparison

In this chapter, a comparison of the scenarios' results is made allowing us to draw some general conclusions to help the strategic decision-making, namely by suggesting the optimal locations to install the new centers and the general costs involved in a functional recovery sorting and storing network that includes besides the RSS center, Task 5.1, the door and windows refitting center, Task 5.2, and a crushing unit, an element of Task 5.3.

The network costs/values depend on the conditions imposed. However, some general conclusions may be drawn about the relative importance of the investment cost relative to the network's operation costs, i.e., the transport and the processing costs. The maintenance and fiscal costs as well as the fiscal costs/incentives are not directly considered in the model due to a lack of available data information. However, future use of the model may easily include the maintenance costs by considering them as part of the processing costs while the fiscal costs/incentives may be associated with the investment costs.

The optimal location to install an RSS center.

All scenarios select Beja as the optimal location to install a single RSS center except for the scenarios that forced the change, by either imposing an increased RCD production in Moura, scenario *B7*; or by imposing a limitation in the installable capacity of the RSS centers, scenario *B8*. Thus, scenario *B7* installs the RSS center in Moura, as expected, while scenario *B8* installs two centers, in Moura and Beja, since the 8000-tonne RSS capacity of Beja is insufficient to accommodate the 13899-tonne of RCDs available for recovery. So it may be concluded that, for the base scenarios of RCD production[1], <u>Beja is the optimal location to install the RSS center.</u>

Network costs/values analysis.

The voucher costs of 90€ per door and 75€ per window paid to incentive the refitting of reusable materials remain the same for all scenarios. The number of 40 doors and 49 windows also remains the same for all scenarios. The optimized costs/values presented in the table below allow for assessing the relative importance of the investment cost compared to the network's operation costs. Thus, assuming a life expectancy of at least 5 years without major maintenance for the installed centers, transport becomes the major cost factor even in the absence of crushing or refitting (scenarios A1 and B1).

[1] <u>These base scenarios were defined by IrRADIARE by email</u>, following a teleconference with the participation of LNEG, APA, ResiAlentejo, and IrRadiare, and after several attempts to get data from the municipal partners either at the teleconferences organized by CIMBAL, or by sending templates (Excel files) via email for all project partners envisaging the reporting of the municipal CDW production data, and by direct email contacts to CIMBAL asking to reinforce, next to municipalities, the need of obtaining and sending the needed data.

Table 12 - The costs/values for all optimized scenarios

	Partial						Total	
	Costs (€)			Value [€]			Cost (€)	Value (€)
	InvCost	TrCost	OpCost	s23	s31	s32	TotCost	TValue
A1	77622	15683	5410				98715	
A2	86872	20123	11996	148000			118991	148000
A3	86872	20129	20436	148000	12000	12000	127437	172000
B1	77622	26689	8661				112972	
B2	86872	31129	15247	148000			133248	148000
B3	86872	33806	19220	237260			139898	237260
B4	86872	48991	22655	237260			158518	237260
B5	86872	31134	23688	148000	12000	12000	141694	172000
B6	86872	33812	27660	237260	12000	12000	148344	261260
B7	86872	44046	30909	277380	12000	12000	161828	301380
B8	173744	38797	30909	277380	12000	12000	243451	301380

For all the scenarios involving at least the production of crushed material, the value of the processed materials surpasses the total network cost thus showing that the installation of an RSS network is profitable. This is a very promising result, but it should be emphasized here that a better set of parameters, maybe more realistic, should be used to validate or dismiss this conclusion.

10.Deliverables

Several attempts to get data from the municipal partners either at the teleconferences organized by CIMBAL, or by sending templates (Excel files) via email for all project partners envisaging the reporting of the municipal CDW production data remain unanswered despite all the efforts made. The same happen in the sequence of direct email contacts with CIMBAL asking to reinforce, next to municipalities, the need of obtaining and sending the needed data.

A preliminary version of this report was sent to all project partners for analysis, feedback, and discussion.

11.Conclusions

This work presents a general mathematical model for the recovery sorting and storage of CDWs and reusable materials with the inclusion of a crushing unit and an off site refitting centre.

Thanks to the independent parametrization it allows it to be used at diverse geographic scales, and use diverse case studies that conform to a basic layout, besides the use of diverse scenarios. It is suited for sensitivity studies of critical factors, to help build coherent strategic decisions, while optimizing CDW recovery and the reuse of materials thus contributing to a more circular economy.

This work showed that for the same installation costs the choice of a location to install the RSS centre depends basically on the transportation costs, that the limitation of the centre's capacity may lead to the installation of additional centres if the quantities of recoverable materials overstep that limit.

Iceland Liechtenstein Norway grants

Annex I – Sets and Parameters of the Case Study

The data presented next are the values used to define the case study. They are common to all the defined scenarios and are the ones proposed to the pertinent project partners for analysis and approval, they constitute the ones that were possible to gather from the published and online data. Table I.1 to Table I.4 present the main sets used by the model.

Sets used

Table I.1 - Set s identifying all states

Element	Name	
s1	Concrete	CDWs Materials
s2	Bricks	CDWs Materials
s3	Tiles and Ceramics	CDWs Materials
s4	Concrete bricks tiles and ceramics Mix	CDWs Materials
s5	Wood	CDWs Materials
s6	Glass	CDWs Materials
s7	Plastic	CDWs Materials
s8	Bituminous mixtures	CDWs Materials
s9	Mixed metals	CDWs Materials
s10	Insulation materials	CDWs Materials
s11	Gypsum-based construction materials	CDWs Materials
s12	CDW containing dangerous substances	CDWs Materials
s13	Mix construction and demolition wastes	CDWs Materials
s14	Reusable roof Tiles	Reusable Materials
s15	Reusable Decorative wall tiles	Reusable Materials
s16	Reusable Decorative floor tiles	Reusable Materials
s17	Reusable Decorative Marble	Reusable Materials
s18	Reusable Decorative Stone	Reusable Materials
s19	Reusable Masonry Stone	Reusable Materials
s20	Reusable Structural wood	Reusable Materials
s21	Reusable Doors	Reusable Materials
s22	Reusable windows	Reusable Materials
s23	Filling Material	Products
s24	Processed Roof Tiles	Products
s25	Processed Decorative wall tiles	Products
s26	Processed Decorative floor tiles	Products
s27	Processed Decorative Marble	Products
s28	Processed Decorative Stone	Products
s29	Processed Masonry Stone	Products
s30	Processed Structural wood	Products
s31	Processed Doors	Products
s32	Processed windows	Products

Table I.2 – Set i of county's names

Element	Name
i1	Aljustrel
i2	Almodôvar
i3	Alvito
i4	Barrancos
i5	Beja
i6	Castro Verde
i7	Cuba
i8	Ferreira do Alentejo
i9	Mértola
i10	Moura
i11	Ourique
i12	Serpa
i13	Vidigueira

Table I.3 – Set k of processing units

Element	Process Name
k0	Dummy process
k1	Recover, sorting, storing
k2	Crushing and sieving
k3	Repair and fitting

Table I.4 – Set r of possible install locations

Element	Name
i5	Beja
i10	Moura
i12	Serpa

To allow the set elements to appear more than once in a variable or equation, a set of aliases is defined, $i = j,\ k = kp,\ s = sp$, with the elements in the same order.

Table I.5 – Processes subsets to specify the adequate set elements in the model's equations

SubSet	Text Explaining the function	SubSet Elements
$kpr(k)$	All the real processes	$\{k1, k2, k3\}$
$krf(k)$	Processes producing crushed stone and Refitted Materials	$\{k2, k3\}$
$kf1(k)$	Processes receiving material from k1	$\{k1, k2, k3\}$
$kf2(k)$	Processes receiving material from k2	$\{k1\}$
$kf3(k)$	Processes receiving material from k3	$\{k1\}$

Table I.6 – Subsets of materials to specify the adequate set elements in the model's equations

SubSet	Text Explaining the function	SubSet Elements
$Rk1(s)$	Subset of input Materials at *k1*	$\{s1,...,s22\}$
$Rk2(s)$	Subset of input Materials at *k2*	$\{s1,...,s4, s13\}$
$Rk3(s)$	Subset of input Materials at *k3*	$\{s21,...,s22\}$
$Ok0(s)$	Subset of processed Materials at *k0*	$\{s1,...,s22\}$
$Ok1(s)$	Subset of processed Materials at *k1* [2]	$\{s1,...,s22, s24,...,s30\}$
$Ok2(s)$	Subset of processed Materials at *k2*	$\{s23\}$
$Ok3(s)$	Subset of processed Materials at *k3*	$\{s31, s32\}$

[2] Some of the materials "processed" at *k1*, namely *s1* to *s13*, may be simply sent to an intermediate storage.

Table I.7 – Subsets of the k raw materials and products s for the mass balance equations

SubSet	Text Explaining the function	SubSet Elements of k	SubSet Elements of s
$Iks(k,s)$	Raw materials *s* of process *k*	$\{k1\}$	$\{Rk1\}$
		$\{k2\}$	$\{Rk2\}$
		$\{k3\}$	$\{Rk3\}$
$Oks(k,s)$	Products *s* of process *k*	$\{k1\}$	$\{Ok1\}$
		$\{k2\}$	$\{Ok2\}$
		$\{k3\}$	$\{Ok3\}$
$k1Sell(s)$	Products stored at k1 that may be sold	$s \in \{s23,...,s32\}$	

Parameter Values

Table I.8 – Dist(i,r), The distances from r(i) to i [km]

	i1	i2	i3	i4	i5	i6	i7	i8	i9	i10	i11	i12	i13
i5	38.3	71.1	37.0	101.0	5.0	23.2	20.0	24.0	53.0	52.1	73.1	32.9	25.1
i10	87.0	118.0	61.7	49.8	52.3	95.1	45.7	69.2	88.5	5.0	109.0	34.0	35.8
i12	65.4	86.2	62.4	76.6	28.0	63.2	46.4	50.6	57.1	34.9	77.5	5.0	43.7

All the materials transportation uses a five-tonne transport

Table I.9 – cp(i,j) Unitary transport cost from i to j [€/ t]

	i1	i2	i3	i4	i5	i6	i7	i8	i9	i10	i11	i12	i13
i5	2.3	4.27	2.22	6.06	0.3	1.39	1.2	1.44	3.18	3.13	4.39	1.97	1.51
i10	5.22	7.08	3.7	2.99	3.14	5.71	2.74	4.15	5.31	0.3	6.54	2.04	2.15
i12	3.92	5.17	3.74	4.6	1.68	3.79	2.78	3.04	3.43	2.09	4.65	0.3	2.62

The model uses a “dummy” **process *k0* installed at each location** to load the materials and send it to the available *k1* processes that are not included in the *L0ik(i,k)* table below for sake of space. All the tables referring to existence parameters use a 1 if it exists and a 0 or a blank if not.

Table I.10 – L0ik(i,k), Locations available to install the processing units k

	k1	k2	k3
i5	1	1	
i10	1	1	1
i12	1	1	

Table I.11 – Lig(k,kp), Allows the transport connections between processing units

	k1	k2	k3
k0	1		
k1	1	1	1
k2	1		
k3	1		

Besides all the *k0* units that are considered pre-installed at each location, the case study also considers a *k3* unit pre-installation at Beja. The corresponding values are written in the parameter *E0ik(i,k).*

Table I.12 – E0ik(i,k), Pre-existent processing centres (1->yes 0->otherwise)

	i1	i2	i3	i4	i5	i6	i7	i8	i9	i10	i11	i12	i13
k0	1	1	1	1	1	1	1	1	1	1	1	1	1
k1													
k2													
k3					1								

Once again, the k0 units have an installation and operation null cost and are not shown in the tables.

Table I.13 – Inv(i,k) Installation Cost for k at i [€]

	k1	k2	k3
i5	77622	9250	0
i10	77622	9250	0
i12	77622	9250	0

Table I.14 – Cop(i,k) Operative Cost for k at i [€/t]

	k1	k2	k3
i5	0.73	0.89	4306.25
i10	0.73	0.89	4306.25
i12	0.73	0.89	4306.25

Table I.15 - v(k,s,sp), Proportion of material at state sp from state s by process k1

k1 Raw Material	*k1* products							
	s13	s14	s15	s16	s17	s18	s19	s20
s14	0.2	0.8						
s15	0.3		0.7					
s16	0.1			0.9				
s17	0.5				0.5			
s18	0.3					0.7		
s19	0.03						0.97	
s20	0.01							0.99

The material at states *s1,..,s13,* and *s21,s22* are not processed by *k1* and are temporarily deposited at the RSS center. They have a transformation factor of 1 in the diagonal elements of the *v(k,s,sp)* matrix, and for sake of space are not presented in this table.

All the remaining *k2,* and *k3* are considered to have no losses or gains in their transformation processes, so they only **have a unitary factor at the diagonal elements on the *v(k,s,sp)* matrix**. The *s23* product material of *k2* is obtained from the *s1,…,s4,* and *s13* input materials while *k3* produces *s31* and *s32* from *s21* and *s22*, respectively.

Annex II – Wide list of optimized scenarios' results

Scenario A1

This scenario A1 uses the quantities of recoverable materials presented for a type A scenario and the demand values of *s23*=0, s*31*=0, and *s32*= 0 [t].

This scenario optimization uses 9734 variables, and the computational resources of 0.11 s to generate and 0.078 s to execute, leading to a minimized total cost of 98715€.

The optimal value is obtained by installing a *k1* unit in an RSS centre at Beja, location *i5*, with the null demand values leading to no additional processing unit being installed to avoid increasing the network's total costs. The network reduction costs occurs by avoiding the transport costs of the reusable and the refitted materials to and from the pre-existing *k3* unit.

The RSS network's individual costs are presented in Table II.1, The voucher costs for doors and windows is shown in Table II.2 while its materials flow is presented in Table II.3 with all the quantities in metric tonnes.

Table II.1 – Optimized network costs for scenario A1

	Partial Costs (€)			Total Cost (€)
	Investment	**Transport**	**Operative**	
A1	77622	15683	5410	98715

Table II.2 – Voucher costs for scenario A1

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.3 – Materials flow in the A1 RSS network (t)

From		To		Materials transported		
				s13	s21	s22
k0	i1	k1	i5	146		
k0	i2	k1	i5	1900		
k0	i3	k1	i5	121		
k0	i5	k1	i5	1682	0.33	0.33
k0	i6	k1	i5	2061		
k0	i8	k1	i5	898		
k0	i9	k1	i5	242		
k0	i10	k1	i5	88	0.33	0.33
k0	i11	k1	i5	239		
k0	i12	k1	i5	32	0.35	0.32
k1	i5	k1	i5	7409	1.00	0.98

This scenario presents only the materials' recovery from all the counties, with no *k1* sorting or cleaning because *a)* the, *s13*, mCDW material recovered demands no further processing by *k1*, *b)* the *k2* crushing unit is not installed, and *c)* the pre-installed *k3* is not used.

The *k1* RSS centre at Beja stores 7409 metric tonnes of mixed construction and demolition wastes, s13, and 40 doors and 49 windows, respectively the equivalent to 1 tonne of doors and 0.98 tonnes of windows to be refitted.

Scenario A2

This scenario uses the quantities of the recoverable material presented for scenario type A and the demand values of *s23*=7400, *s31*=0, and *s32*= 0.

The optimization of this scenario uses 9732 variables, and the computational resources of 0.125s to generate and 0.062 s to execute, leading to a minimized total cost of 118991 €

The optimal value is obtained by installing the units *k1* and *k2* at an RSS centre at Beja, the model's location *i5*, with the *s31* and *s32* null demand values imposing that **the existing refitting unit is not used** to avoid the unnecessary additional network costs for transporting to and from *k3* and process the corresponding reusable materials.

The RSS network's individual costs are presented in Table II.4, the voucher costs for doors and windows are shown in Table II.5, and the value for the products is shown in Table II.6 while its materials flow is presented in Table II.7 with all the quantities in metric tonnes.

Table II.4 – Optimized network costs for scenario A2

	Partial Costs (€)			Total Cost (€)
	Investment	Transportation	Operational	Total
A2	86872	20123	11996	118991

Table II.5 – Voucher costs for scenario A2

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.6 – Optimized network value for the A2 products

K1 Location	Value [€]	
	s23	TValue
i5	148000	148000

Table II.7 – Materials flow in the A2 RSS network (t)

From		To		Quantity of materials transported			
				s13	s21	s22	s23
k0	i1	k1	i5	146			
k0	i2	k1	i5	1900			
k0	i3	k1	i5	121			
k0	i5	k1	i5	1682	0.325	0.33	
k0	i6	k1	i5	2061			
k0	i8	k1	i5	898			
k0	i9	k1	i5	242			
k0	i10	k1	i5	88	0.325	0.33	
k0	i11	k1	i5	239			
k0	i12	k1	i5	32	0.35	0.32	
k1	i5	k1	i5	7409	1	0.98	
k1	i5	k2	i5	7400			
k2	i5	k1	i5				7400

The A2 network now sends 7400 tonnes of mCDW recovered material, s13, to receive and store the same quantity of crushed material s23 from *k1*, thus leaving stored 9 tonnes of s13. The number of 40 doors and 49 windows to refit remain stored since Beja's *k3* refitting centre is not used due to the absence of refitted products demand.

Table II.8 – A2 network's materials storage

Location	Types of materials stored			
	s13	s21	s22	s23
i5	9	40	49	7400

Scenario A3

This scenario uses the recoverable material values presented for scenario type A and the demand values of *s23*=7400, *s31*=1, and *s32*= 0.95.

This scenario optimization uses 9732 variables, and the computational resources of 0.094 s to generate and 0.109 s to execute, leading to a minimized total cost of 127437 €

The A3 optimal solution **installs a *k1* and a *k2* unit in an RSS centre at Beja**, the location *i5*, **and uses the available *k3* pre-installed unit at Beja**. The RSS network's individual costs are shown in Table II.9, the voucher costs are presented in Table II.10, the network values are presented in Table II.11, and its materials flow is presented in Table II.12 with all the quantities in metric tonnes.

Table II.9 – Optimized network costs for scenario A3

	Partial Costs (€)			Total Cost (€)
	Investment	Transportation	Operational	Total
A3	86872	20129	20436	127437

Table II.10 – Voucher costs for scenario A3

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.11 – Optimized network value for the A3 products

K1 Location	Value [€]			
	s23	s31	s32	TValue
i5	148000	12000	12000	172000

Table II.12 – Materials flow in the A3 RSS network (t)

From		To		Materials transported					
				s13	s21	s22	s23	s31	s32
k0	i1	k1	i5	146					
k0	i2	k1	i5	1900					
k0	i3	k1	i5	121					
k0	i5	k1	i5	1682	0.33	0.33			
k0	i6	k1	i5	2061					
k0	i8	k1	i5	898					
k0	i9	k1	i5	242					
k0	i10	k1	i5	88	0.33	0.33			
k0	i11	k1	i5	239					
k0	i12	k1	i5	32	0.35	0.32			
k1	i5	k1	i5	7409	1.00	0.98			
k1	i5	k2	i5	7400					
k1	i5	k3	i5		1	0.96			
k2	i5	k1	i5				7400		
k3	i5	k1	i5					1	0.96

The A3 network material storage now receives 7400 tonnes of *s23* material crushed by *k2* and maintains stored 9 tonnes of *s13* and a single *s22* reusable window, while *k3* sends for storage 40 *s31* refitted doors and 48 *s32* refitted windows as shown in Table II.13.

Table II.13 – A3 network's materials storage

Location	Types of materials stored				
	s13	s22	s23	s31	s32
i5	9	1	7400	40	48

Scenario B1

This scenario uses the recoverable material values presented for scenario type B and the demand values of s23=0, s31=0, and s32= 0.

This scenario optimization uses 9732 variables, and the computational resources of 0.094 s to generate and 0.125 s to execute, leading to a minimized total cost of 112972€

The optimal value is obtained by installing a *k1* unit in an RSS centre at Beja, the location *i5*, **not installing a *k2* unit, and not using the available *k3* pre-installed unit at Beja**. The RSS network's individual costs are shown in Table II.14, the voucher costs are shown in Table II.15, the network values are presented in Table II.16, and its materials flow is presented in Table II.17 with all the quantities in metric tonnes.

Table II.14 – Optimized network costs for scenario B1

	Partial Costs (€)			Total Cost (€)
	Investment	**Transportation**	**Operational**	**Total**
RSSmodel	77622	26689	8661	112972

Table II.15 – Voucher costs for scenario B1

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.16 – Optimized network value for the B1 products

K1 Location	Value [€]			
	s23	s31	s32	TValue
i5	148000	12000	12000	172000

Table II.17 – Materials flow in the B1 RSS network (t)

From		To		Materials transported		
				s13	s21	s22
k0	i1	k1	i5	350		
k0	i2	k1	i5	1910		
k0	i3	k1	i5	1323		
k0	i4	k1	i5	186		
k0	i5	k1	i5	1810	0.325	0.33
k0	i6	k1	i5	2221		
k0	i8	k1	i5	1020		
k0	i9	k1	i5	1526		
k0	i10	k1	i5	294	0.325	0.33
k0	i11	k1	i5	429		
k0	i12	k1	i5	750	0.35	0.32
k0	i13	k1	i5	44		
k1	i5	k1	i5	11863	1	0.98

The B1 network maintains stored 11863 tonnes of *s13* and 40 reusable doors, *s31,* and 49 reusable windows, *s31,* to be refitted that correspond to the values shown in Table II.18.

Table II.18 – B1 network's materials storage

Location	Types of materials stored		
	s13	*s21*	*s22*
i5	11863	40	49

Scenario B2

This scenario uses the recoverable material values presented for scenario type B and the demand values of s23=7400, s31=0, and s32= 0.

The optimization of this scenario uses 9732 variables, and the computational resources of 0.11 s to generate and 0.157 s to execute, leading to a minimized total cost of 133248 €

The optimal value is obtained by installing units *k1* and *k2* at an RSS centre at Beja, location *i5*, with the *s31* and *s32* null demand values imposing that **the existing refitting unit is not used** to avoid unnecessary additional network costs. The RSS network's individual costs are presented in Table II.19, the voucher costs are shown in Table II.20, the network values are presented in Table II.21, and its materials flow is presented in Table II.22 with all the quantities in metric tonnes.

Table II.19 – Optimized network costs for scenario B2

	Partial Costs (€)			Total Cost (€)
	Investment	Transportation	Operational	Total
B2	86872	31129	15247	133248

Table II.20 – Voucher costs for scenario B2

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.21 – Optimized network value for the B2 products

K1 Location	Value [€]	
	s23	TValue
i5	148000	148000

Table II.22 – Materials flow in the B2 RSS network (t)

From		To		Quantity of materials transported			
				s13	s21	s22	s23
k0	i1	k1	i5	350			
k0	i2	k1	i5	1910			
k0	i3	k1	i5	1323			
k0	i4	k1	i5	186			
k0	i5	k1	i5	1810	0.325	0.33	
k0	i6	k1	i5	2221			
k0	i8	k1	i5	1020			
k0	i9	k1	i5	1526			
k0	i10	k1	i5	294	0.325	0.33	
k0	i11	k1	i5	429			
k0	i12	k1	i5	750	0.35	0.32	
k0	i13	k1	i5	44			
k1	i5	k1	i5	11863	1	0.98	
k1	i5	k2	i5	7400			
k2	i5	k1	i5				7400

The B2 network material storage now processes 7400 tonnes of s13 to the same quantity of crushed material s23 thus leaving stored 4463 tonnes of s13. The number of 40 doors and 49 windows to refit remains the same as in the A1 base scenario as shown in Table II.23.

Table II.23 – B2 network's materials storage

Location	Types of materials stored			
	s13	*s21*	*s22*	*s23*
i5	4463	40	49	7400

Scenario B3

This scenario uses the recoverable material values presented for scenario type B and the demand values of <u>s23=11863</u>, s31=0, and s32= 0.

This scenario optimization uses 9734 variables, and the computational resources of 0.140 s to generate and 0.093 s to execute, leading to a minimized total cost of 139898 €

Despite the increase in the *s23* demand, the B3 optimal value is again obtained by installing a *k1* and a *k2* unit in an RSS centre at Beja, the location *i5*, **and by not using the available *k3* pre-installed unit** at Beja. The RSS network's individual costs are shown in Table II.24, the voucher costs are shown in Table II.25, the network values are presented in Table II.26, and its materials flow is presented in Table II.27 with all the quantities in metric tonnes.

Table II.24 – Optimized network costs for scenario B3

	Partial Costs (€)			Total Cost (€)
	Investment	Transportation	Operational	Total
RSSmodel	86872	33806	19220	139898

Table II.25 – Voucher costs for scenario B3

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.26 – Optimized network value for the B3 products

K1 Location	Value [€]	
	s23	TValue
i5	237260	

Table II.27 – Materials flow in the B3 RSS network (t)

From		To		Quantity of materials transported			
				s13	*s21*	*s22*	*s23*
k0	i1	k1	i5	350			
k0	i2	k1	i5	1910			
k0	i3	k1	i5	1323			
k0	i4	k1	i5	186			
k0	i5	k1	i5	1810	0.325	0.33	
k0	i6	k1	i5	2221			
k0	i8	k1	i5	1020			
k0	i9	k1	i5	1526			
k0	i10	k1	i5	294	0.325	0.33	
k0	i11	k1	i5	429			
k0	i12	k1	i5	750	0.35	0.32	
k0	i13	k1	i5	44			
k1	i5	k1	i5	11863	1	0.98	
k1	i5	k2	i5	11863			
k2	i5	k1	i5				11863

The B3 network material storage now receives 11863 tonnes of *s23* material crushed by *k2* and stores 40 *s21* doors and 48 *s22* windows able to be refitted as shown in Table II.28.

Table II.28 – B3 network's materials storage

Location	Types of materials stored		
	s21	*s22*	*s23*
i5	40	49	11863

Scenario B4

The main purpose of this scenario is to test for a change in the *k1* install location. Therefore, it uses the recoverable material values presented for **a scenario of type B <u>with a 5000 tonne of s13 made available at location i10</u>, and the demand values of *s23*=11863, *s31*=0, and *s32*= 0**.

This scenario optimization uses 9732 variables, and the computational resources of 0.125s to generate and 0.063s to execute, leading to a minimized total cost of 158518 €

The increase in the *s13* mixed CDW leads, as expected, to an optimal value now obtained by <u>installing a *k1* and a *k2* unit in an RSS centre at Moura</u>, the location *i10*, **and by not using the available *k3* pre-installed unit at Beja**. The RSS network's individual costs are shown in Table II.29, the voucher costs are shown in Table II.30, the network values are presented in Table II.31, and its materials flow is presented in Table II.32 with all the quantities in metric tonnes.

Table II.29 – Optimized network costs for scenario B4

	Partial Costs (€)			Total Cost (€)
	Investment	Transportation	Operational	Total
B4	86872	48991	22655	158518

Table II.30 – Voucher costs for scenario B4

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.31 – Optimized network value for the B4 products

K1 Location	Value [€]	
	s23	TValue
I10	237260	237260

Table II.32 – Materials flow in the B4 RSS network (t)

From		To		Quantity of materials transported			
				s13	*s21*	*s22*	*s23*
k0	i1	k1	i10	350			
k0	i2	k1	i10	1910			
k0	i3	k1	i10	1323			
k0	i4	k1	i10	186			
k0	i5	k1	i10	1810	0.325	0.33	
k0	i6	k1	i10	2221			
k0	i8	k1	i10	1020			
k0	i9	k1	i10	1526			
k0	i10	k1	i10	5000	0.325	0.33	
k0	i11	k1	i10	429			
k0	i12	k1	i10	750	0.35	0.32	
k0	i13	k1	i10	44			
k1	i10	k1	i10	16569	1	0.98	
k1	i10	k2	i10	11863			
k2	i10	k1	i10				11863

The B4 network as shown in Table II.33 now stores 4706 tonnes of *s13* material and receives 11863 tonnes of *s23* material crushed by *k2* while storing 40 *s21* doors and 48 *s22* windows that are suitable for refitting.

Table II.33 – B4 network's materials storage

Location	Types of material stored			
	s13	s21	s22	s23
i10	4706	40	49	11863

Scenario B5

This scenario uses the recoverable material values of **scenario type B, and the demand values of *s23*=7400, *s31*=1, and *s32*= 0.96**.

This scenario optimization uses 9732 variables, and the computational resources of 0.109 s to generate and 0.078 s to execute, leading to a minimized total cost of 141694 €

B5 scenario optimal value is now obtained by installing a *k1* and a *k2* unit in an RSS centre at Beja, the location *i5*, **and by using the available *k3* pre-installed unit at Beja**. The RSS network's individual costs are shown in Table II.34, the voucher costs are presented in Table II.35, the network values are presented in Table II.36, and its materials flow is presented in Table II.*37*Table II.37

Table II.34 – Optimized network costs for scenario B5

	Partial Costs (€)			Total Cost (€)
	Investment	**Transportation**	**Operational**	**Total**
B5	86872	31134	23688	141694

Table II.35 – Voucher costs for scenario B5

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.36 – Optimized network value for the B5 products

K1 Location	Value [€]			
	s23	s31	s32	TValue
i5	148000	12000	12000	172000

Table II.37 – Materials flow in the B5 RSS network (t)

From		To		Quantity of materials transported					
				s13	s21	s22	s23	s31	s32
k0	i1	k1	i5	350					
k0	i2	k1	i5	1910					
k0	i3	k1	i5	1323					
k0	i4	k1	i5	186					
k0	i5	k1	i5	1810	0.325	0.33			
k0	i6	k1	i5	2221					
k0	i8	k1	i5	1020					
k0	i9	k1	i5	1526					
k0	i10	k1	i5	294	0.325	0.33			
k0	i11	k1	i5	429					
k0	i12	k1	i5	750	0.35	0.32			
k0	i13	k1	i5	44					
k1	i5	k1	i5	11863	1	0.98			
k1	i5	k2	i5	7400					
k1	i5	k3	i5		1	0.96			
k2	i5	k1	i5				7400		
k3	i5	k1	i5					1	0.96

As shown in Table II.38, the B5 network now stores 4463 tonnes of *s13* material and one window to be refitted receiving the demanded 7400 tonnes of *s23* material crushed by *k2* while storing 40 *s21* doors and 48 *s22* windows refitted by Beja's *k3* unit.

Table II.38 – B5 network's materials storage

Location	Types of material stored				
	s13	s22	s23	s31	s32
i5	4463	1	7400	40	48

Scenario B6

This scenario uses the recoverable material values of **scenario type B, and the demand values of *s23*=11863, *s31*=1, and *s32*= 0.96**.

This scenario optimization uses 9732 variables, and the computational resources of 0.109 s to generate and 0.125 s to execute, leading to a minimized total cost of 148344 €

B6 scenario optimal value is now obtained by installing a *k1* and a *k2* unit in an RSS centre at Beja, the location *i5*, **and by using the available *k3* pre-installed unit at Beja**. The RSS network's individual costs are shown in Table II.39, the voucher costs are presented in Table II.40, the network values are presented in Table II.41, and its materials flow is presented in Table II.42 with all the quantities in metric tonnes.

Table II.39 – Optimized network costs for scenario B6

	Partial Costs (€)			Total Cost (€)
	Investment	Transportation	Operational	Total
B5	86872	33812	27660	148344

Table II.40 – Voucher costs for scenario B6

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.41 – Optimized network value for the B6 products

K1 Location	Value [€]			
	s23	s31	s32	TValue
i5	148000	12000	12000	172000

Table II.42 – Materials flow in the B6 RSS network (t)

From		To		Quantity of materials transported					
				s13	s21	s22	s23	s31	s32
k0	i1	k1	i5	350					
k0	i2	k1	i5	1910					
k0	i3	k1	i5	1323					
k0	i4	k1	i5	186					
k0	i5	k1	i5	1810	0.325	0.33			
k0	i6	k1	i5	2221					
k0	i8	k1	i5	1020					
k0	i9	k1	i5	1526					
k0	i10	k1	i5	294	0.325	0.33			
k0	i11	k1	i5	429					
k0	i12	k1	i5	750	0.35	0.32			
k0	i13	k1	i5	44					
k1	i5	k1	i5	11863	1	0.98			
k1	i5	k2	i5	11863					
k1	i5	k3	i5		1	0.96			
k2	i5	k1	i5				11863		
k3	i5	k1	i5					1	0.96

As shown in Table II.43, the B6 network now stores 4463 tonnes of *s13* material and one window to be refitted receiving the demanded 7400 tonnes of *s23* material crushed by *k2* while storing 40 *s21* doors and 48 *s22* windows refitted by Beja's *k3* unit.

Table II.43 – B6 network's materials storage

Location	Types of material stored			
	s22	s23	s31	s32
i5	1	11863	40	48

Scenario B7

This scenario uses the recoverable material values of **scenario type B <u>with a 2300 tonne of s13 now available at location i10</u>, and the demand values of *s23*=11863, *s31*=1, and *s32*= 0.96**.

B7 optimization uses 9734 variables, and the computational resources of 0.125 s to generate and 0.156 s to execute, leading to a minimized total cost of 161828 €

B7 scenario optimal value is now obtained by installing a *k1* and a *k2* unit in an RSS centre at Moura, location *i10*, **and by using the available *k3* pre-installed unit at Beja**. The RSS network's individual costs are shown in Table II.44, the voucher costs are presented in Table II.45, the network values are presented in Table II.46, and its materials flow is presented in Table II.*47.*

Table II.44 – Optimized network costs for scenario B7

	Partial Costs (€)			Total Cost (€)
	Investment	Transportation	Operational	Total
B5	86872	44046	30909	161828

Table II.45 – Voucher costs for scenario B7

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.46 – Optimized network value for the B7 products

K1 Location	Value [€]			
	s23	s31	s32	TValue
i10	277380	12000	12000	301380

Table II.47 – Materials flow in the B7 RSS network (t)

From		To		Quantity of materials transported [t]					
				s13	s21	s22	s23	s31	s32
k0	i1	k1	i10	350					
k0	i2	k1	i10	1910					
k0	i3	k1	i10	1323					
k0	i4	k1	i10	186					
k0	i5	k1	i10	1810	0.325	0.33			
k0	i6	k1	i10	2221					
k0	i8	k1	i10	1020					
k0	i9	k1	i10	1526					
k0	i10	k1	i10	2300	0.325	0.33			
k0	i11	k1	i10	429					
k0	i12	k1	i10	750	0.35	0.32			
k0	i13	k1	i10	44					
k1	i10	k1	i10	13869	1	0.98			
k1	i10	k2	i10	13869					
k1	i10	k3	i5		1	0.96			
k2	i10	k1	i10				13869		
k3	i5	k1	i10					1	0.96

As shown in Table II.48, the B7 network now stores 4463 tonnes of *s13* material and one window to be refitted receiving the demanded 7400 tonnes of *s23* material crushed by *k2* while storing 40 *s21* doors and 48 *s22* windows refitted by Beja's *k3* unit.

Table II.48 – B7 network's materials storage

Location	Types of material stored			
	s22	s23	s31	s32
i10	1	13869	40	48

Scenario B8

This scenario uses the recoverable material values of **scenario type B and upper limits the k1 capacity to 8000 tonnes, and the demand values of *s23*= 13869, *s31*=1, and *s32*= 0.96**.

This scenario optimization use 9732 variables, and the computational resources of 0.094 s to generate and 0.094 s to execute, leading to a minimized total cost of 243451 €

B8 scenario optimal value is now obtained by installing a *k1* and a *k2* unit in RSS centres at Beja and Moura, **and by using the available *k3* pre-installed unit at Beja**. The RSS network's individual costs are shown in Table II.49, the cost of the vouchers is shown in Table II.50, the network product values are shown in Table II.51 and its materials flow is presented in Table II.52 with all the quantities in metric tonnes.

Table II.49 – Optimized network costs for scenario B8

	Partial Costs (€)			Total Cost [€]
	InvCost	TrCost	OpCost	TotCost
B8	173744	38797	30909	243451

Table II.50 – Voucher costs for B8 products

	Voucher Cost [€]		
	s21	s22	TVouch
k1	3600	3675	7275

Table II.51 – Optimized network value for B8 products

k1 location	Product Value [€]			
	s23	s31	s32	TValue
i5	159961	12000	12000	183961
i10	117419			117419

Table II.52 – Materials flow in the B8 RSS network (t)

From		To		Materials transported					
				s13	s21	s22	s23	s31	s32
k0	i1	k1	i10	350					
k0	i2	k1	i10	1910					
k0	i3	k1	i5	1323					
k0	i4	k1	i10	186					
k0	i5	k1	i5	1810	0.33	0.33			
k0	i6	k1	i5	2221					
k0	i8	k1	i5	1020					
k0	i9	k1	i5	401					
k0	i9	k1	i10	1125					
k0	i10	k1	i5		0.33	0.31			
k0	i10	k1	i10	2300		0.02			
k0	i11	k1	i5	429					
k0	i12	k1	i5	750	0.35	0.32			
k0	i13	k1	i5	44					
k1	i5	k1	i5	7998.04	1	0.96			
k1	i5	k2	i5	7998.04					
k1	i5	k3	i5		1	0.96			
k1	i10	k1	i10	5870.96		0.02			
k1	i10	k2	i10	5870.96					
k2	i5	k1	i5				7998.04		
k2	i10	k1	i10				5870.96		
k3	i5	k1	i5					1	0.96

The limitation to 8000 tonnes of the maximal allowed capacity in face of the quantity of material to be recovered, namely of 13869 tonnes of mCDW, 1 tonne of doors, and 0.98 tonnes of windows, impose the installation of two RSS centres, one in Beja with the maximal allowed capacity and another in Moura with a total capacity 5870.98 tonnes in Moura.

As shown in Table II.53, the B8 network now stores one window to be refitted at Moura which corresponds to a weight of 0.02 tonnes. This is particularly interesting because it corresponds to one of the recoverable windows available in Moura that does not need to be processed to fulfill the demand and as such stays in the Moura RSS centre.

Table II.31 also shows the demanded 13869 tonnes of *s23* crushed material, are obtained by processing 7998.04 tonnes at the *k2* unit of Beja and 5870.96 at the *k2* unit of Moura. That table also shows that the 40 *s21* doors and 48 *s22* windows needed to fulfill the demand were refitted by Beja's *k3* unit and stored at the RSS centre of Beja.

Table II.53 – B8 network's materials storage

Location	Types of materials stored			
	s22	s23	s31	s32
i5		7998.04	40	48
i10	1	5870.96		

Annex IV - Mathematical Model

The core of the mathematical model presented below describes a recovery, sorting, and storing network for RCDs and RMs, with the eventual inclusion of crushing and RM recovery units.

Next are presented the sets and variables of the model used, followed by the presentation of the equations.

Sets & subsets used

s	-	Specifies the types of CDWs, Reusable Materials, Intermediate and Final Products,	
k	-	Identifies the possible types of units/centers that process the network materials,	
i	-	Identifies the network's locations,	
j	-	Defines a set *j* identical to the set *i* by using	$alias(i,j)$
kp	-	Defines a set *kp* identical to the set *k* by using	$alias(k,kp)$
sp	-	Defines a set *sp* identical to the set *s* by using	$alias(s,sp)$
$r(i)$	-	Possible locations for the network Recover Sorting and Storing centers	
$kpr(k)$	-	Identifies all the *k* real processes,	$kpr(k) \mid k \in \{k1,k2,k3\}$
$krf(k)$	-	Identifies all the processes *k2* and *k3*,	$krf(k) \mid k \in \{k2,k3\}$
$kf1(k)$	-	All the processes receiving material from *k1*,	$kf1(k) \mid k \in \{k1,k2,k3\}$
$kf2(k)$	-	All the processes receiving material from *k2*,	$kf2(k) \mid k \in \{k1\}$
$kf3(k)$	-	All the processes receiving material from *k3*,	$kf3(k) \mid k \in \{k1\}$
$Rk1(s)$	-	A subset of input materials at *k1*,	$Rk1(s) \mid s \in \{s1,s2,\ldots,s22\}$
$Rk2(s)$	-	A subset of input materials at *k2*,	$Rk2(s) \mid s \in \{s1,\ldots,s4,s13\}$
$Rk3(s)$	-	A subset of input materials at *k3*,	$Rk3(s) \mid s \in \{s21,s22\}$
$Ok0(s)$		Materials *s* loaded by the dummy process *k0*,	$Ok0(s) \mid s \in \{s1,s2,\ldots,s22\}$
$Ok1(s)$	-	A subset of materials processed at *k1*,	$Ok1(s) \mid s \in \{s1,s2,\ldots,s22\}$
$Ok2(s)$	-	A subset of materials processed at *k2*,	$Ok2(s) \mid s \in \{s23\}$
$Ok3(s)$	-	A subset of materials processed at *k3*,	$Ok3(s) \mid s \in \{s31,s32\}$
$k1Sell(s)$	-	A Subset to identify k1 stored products to sell	$k1Sell(s) \mid s \in \{s23,\ldots,s32\}$
$iks(k,s)$	-	Processes' raw materials	$iks(k,s)\ \forall k,s \left\| \begin{Bmatrix} k \in \{k0\} \Rightarrow s \in Rk1(s), \\ k \in \{k1\} \Rightarrow s \in Rk1(s), \\ k \in \{k2\} \Rightarrow s \in Rk2(s), \\ k \in \{k3\} \Rightarrow s \in Rk3(s) \end{Bmatrix} \right.$

$oks(k,s)$ - Processes' raw materials

$$oks(k,s)\ \forall k,s \left| \begin{cases} k \in \{k0\} \Rightarrow s \in Ok0(s), \\ k \in \{k1\} \Rightarrow s \in Ok1(s), \\ k \in \{k2\} \Rightarrow s \in Ok2(s), \\ k \in \{k3\} \Rightarrow s \in Ok3(s) \end{cases} \right.$$

Binary variables used

$Yrsc(k,i)$ - A binary variable to define if the units/centers *k* are installed at *r* (1->Yes, 0->No)

Positive variables

$CIik(i,k)$ - Investment cost to install at *r(i)* the process *k*

$Ck0(i)$ - Transport Loading Cost at *i* (by process *k0*)

$Ck01(i,j)$ - Transport Cost for materials from k0 at *i* to k1 at *j*

$Ck1n(i,j)$ - Transport Cost for materials of k1 at *i* to and from k2 or k3 at *j*

$CPk(r,k,s)$ - Cost for processing materials *s* in *k* at location *i*

$Capkjs(k,i,s)$ - Total capacity installed for processing *s* at *i* by *k*

$Capkj(k,i)$ - Total capacity installed for process *k* at *i*

$CapTk(k)$ - Installed capacity of process *k*

$Q(s,i,j,k,kp)$ - Quantity of materials *s* transported to *k* at *i* from *kp* at *j*

$QsRM(s,i)$ - Quantity of Raw Materials (RM) arriving (at *k1*) at *i*

$QsCDW(s,i)$ - Quantity of CDW arriving (at *k1*) at *i*

$X(s,r,k)$ - Quantity of intermediate products *s* stored on *k1* at *r*

$Xf(s,r,k)$ - Final quantity of products *s* stored on *k1* at *r*

$QTstor(k,r)$ - Total quantity of materials and products stored on *k1* at *r*

$QsTo(s)$ - Total quantity of products *s* available

$CInv$ - Total Installation Costs for the Processing Units

$CostTr$ - Total Transportation Cost

$CostPr$ - Total Processing Cost

$CostVouch(s)$ - Cost of the vouchers for each Reusable Material *s*

$TCVoucher$ - Total Cost of all the vouchers (for all the Reusable Materials)

$Valuesk(s,k)$ - Value of the reusable products s at k

$Valuesk(s)$ - Value of the reusable products s

$Value$ - Value of the reusable products

Real Variables

$Cost$	- Total Cost
$Profit$	- Profit from selling the reusable products

Equations

This section presents the equations of the model grouped by their function. All equations are preceded by an explanatory text and if needed may be followed by a note or a supplementary explanation.

To help understand the mathematical model described by the equations that follow, an illustration of the network's flow for representative materials/products is presented in the figure below.

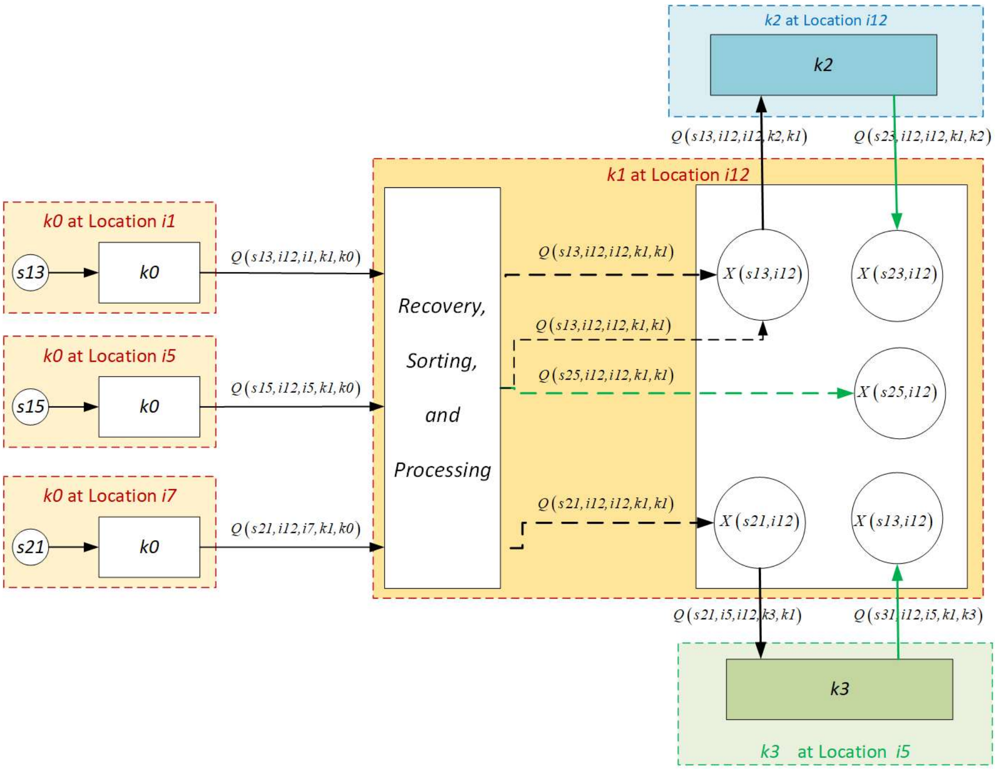

Figure III.1 - An illustration of the network's flow for a set of typical materials/products.

Note: *the positive variable* $Q(s,i,j,k,kp)$ *is a mathematical representation of the quantity of material s flowing from process kp at location j to process k at location i. The set element s identifies the material type of a product of kp that constitutes an input of k.*

To avoid wasting computational time at the pre-solve phase of the optimization process, a lesser number of null equation's instantiations leading to a row of zeros in the matrix representation is side-stepped by imposing conditions that do allow the equations instantiation only when they are verified.

Imposing the layout and the Logical conditions

The equation (1) assumes the pre-existing units or centers *k* at node *i*, specified by the parameter $E0ik(i,k)$ while allowing for further installation if needed:

$$Yrsc(k,i) \geq E0ik(i,k) \quad \forall i,k \;\; | \; E0ik(i,k)=1 \tag{1}$$

Equation (2) constrains the total number of processing units of type *k* installed to a predefined upper limit $Nk(k)$

$$\sum_i Yrsc(k,i) \leq Nk(k) \qquad \forall k \tag{2}$$

Defining and constraining the capacity values of the processes

The equation (3) defines the capacity for loading the existing quantity of material of type *s* at *i*, $Q0(i,s)$, by a "dummy" process *k0*.

$$Capkjs(k,i,s) = Q0(i,s) \qquad \forall\; k,i,s \;\left|k \;|k \in \{k0\}, Q0(i,s) > 0\right. \tag{3}$$

The equation (4) defines the capacity to process the input materials *s* of *k1* at *r* by using the quantity of *s* arriving from the *k0* processes at all locations *i*, $Q(s,r,i,k,kp)$ [3].

$$Capkjs(k,i,s) = \sum_{j\;|Q0(j,s)>0} \; \sum_{kp\;|kp \in \{k0\}} Q(s,i,j,k,kp) \qquad \forall k,i,s \left| \begin{matrix} k \in \{k1\} \\ i \;|i \in \{r\} \end{matrix} \right. \tag{4}$$

The equation (5) defines[4] the capacity for processing the input material *s* by *k2* and *k3* at *i*.

$$Capkjs(k,i,s) = \sum_{kp \left| \begin{matrix} kp \in \{k1\} \\ Lig(kp,k)=1 \end{matrix} \right.} \; \sum_{r\;|L0ik(r,kp)=1} Q(s,i,r,k,kp) \quad \forall k,i,s \left| \begin{matrix} k \in \{k2,k3\},\; i \in \{r\}, \\ s \in \{iks(k,s)\}, \\ L0ik(i,k)=1 \end{matrix} \right. \tag{5}$$

Equation (6) computes the capacity of k at i

$$Capkj(k,i) = \sum_s Capkjs(k,i,s) \qquad \forall k,i \tag{6}$$

Equation (7) Inhibits sending refitted Raw Materials s to an inexistent k1 at i.

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[3] It is assumed that no utilities are added while processing the input material. If that assumption is false consider revising this and the following equations accordingly.

[4] Note: The input parameter *L0ik(i,k)* is used to limit the instantiation of the equation, *i.e.*, to limit the writing of a particular occurrence of the equation to the cases where the respective elements *r, k* verify to the condition *L0ik(i,k)* >0.

$$Capkj(k,i) \geq \sum_{kp \,|\, kp\in\{k2,k3\},\, Lig(kp,k)=1} \; \sum_{j \,|\, j\in\{r\},\, L0ik(j,kp)} \; \sum_{s \,|\, s\in\{iks(s)\}} Q(s,i,j,k,kp)=1 \qquad \forall k,i \,\Big|\, k\in\{k1\}, i\in\{r\}, L0ik(i,k)=1 \tag{7}$$

Equation (8) constrains the capacity of process *k* at *r* to the predefined upper limit $Capmax(r,k)$.

$$Capkj(k,i) \leq Capmax(i,k)\cdot Yrsc(k,i) \qquad \forall\; k,i \;|Capmax(i,k)>0 \tag{8}$$

Forcing the processing of all CDWs and RMs, and the production of the demanded products

Equation (9) ensures that all RCD and Reusable materials *s* produced at location *i* are sent by a dummy process *k0* to the existing processing units *k1* at location *r*.

$$\sum_{kp \,|\, kp\in\{k1\},\, Lig(k,kp)=1} \; \sum_{r \,|\, Loik(r,kp)=1} Q(s,r,i,kp,k) = Q0(i,s) \quad \forall i,k,s \,\Big|\, k\in\{k0\},\, s\in\{Rk1\},\, Q0(i,s)>0 \tag{9}$$

Equation (10) ensures that the quantity of crushed product *s* satisfies the demand[5].

$$\sum_{k \,|\, k\in\{k1\}} \; \sum_{kp \,|\, kp\in\{k2\}} \; \sum_{r \,|\, L0ik(r,kp)=1} Q(s,r,r,k,kp) = Dem(s) \qquad \forall s \,\Big|\, s\in\{Ok2\},\, Dem(s)>0 \tag{10}$$

Equation (11) guarantees that the refitted doors and windows satisfy the demand[6].

$$\sum_{k \,|\, k\in\{k1\}} \; \sum_{kp \,|\, kp\in\{k3\},\, kp\in\{oks(kp,s)\},\, Lig(kp,k)=1} \; \sum_{r \,|\, L0ik(r,kp)=1} \; \sum_{i \,|\, i\in\{r\},\, L0ik(i,kp)=1} Q(s,r,i,k,kp) = Dem(s) \;\; \forall s \,\Big|\, s\in\{Ok3\},\, Dem(s)>0 \tag{11}$$

Performing the mass balance for the processes

Equation (12) performs the mass balance for materials *s* processed by the existing *k1* located at *r* from its input material *sp* sent from *k0* at i.

$$Q(s,r,r,k,k) = \sum_{kp \,|\, kp\in\{k0\},\, Lig(kp,k)=1} \; \sum_{i} \; \sum_{sp \,|\, sp\in iks(k,sp),\, Q0(i,sp)>0} v(k,sp,s)\cdot Q(sp,r,i,k,kp) \qquad \forall r,k,s \,\Big|\, k\in\{k1\},\, s\in\{Ok1\} \tag{12}$$

Equation (13) performs the mass balance for the material *s* processed on the existing *k2* located at *r*

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[5] The presence of the condition $L0ik(r,kp)=1$ in the *r* summation guarantees that no material is sent to any *k1* unit unless the corresponding *k2* unit is allowed to be installed at location *r* by the parametrization.
[6] The same as in the previous note for unit *k3* at location *i*.

$$\left[\begin{array}{l} Q(s,r,r,kp,k) = \\ \sum_{sp \,|\, sp \in \{iks(k,sp)\},\ v(k,sp,s)>0} v(k,sp,s) \cdot Q(sp,r,r,k,kp) \end{array}\right] \quad \forall r,k,kp,s \,\middle|\, \begin{array}{l} s \in \{oks(k,s)\}, \\ k \in \{k2\}, \\ kp \in \{k1\}, \\ Lig(k,kp)=1 \end{array} \tag{13}$$

Equation (14) performs the mass balance for the material *s* processed on the existing *k3* located at *r*

$$\left[\begin{array}{l} Q(s,r,i,kp,k) = \\ \sum_{sp \,|\, sp \in \{iks(k,sp)\},\ v(k,sp,s)>0} v(k,sp,s) \cdot Q(sp,i,r,k,kp) \end{array}\right] \quad \forall i,r,k,kp,s \,\middle|\, \begin{array}{l} k \in \{k3\},\ kp \in \{k1\}, \\ s \in \{oks(k,s)\}, \\ i \in \{r\},\ Lig(kp,k)=1 \end{array} \tag{14}$$

Assessing the quantity of material stored at the RS centres

Equation (15) assesses the quantity of intermediate material[7] s stored in *k1* located at *r* after sending materials for crushing by *k2* at *r* and for refitting by *k3* at *i*.

$$X(s,r,k) = \left[\begin{array}{l} Q(s,r,r,k,k) \quad - \\ \sum_{kp \,|\, kp \in \{k2\},\ iks(kp,s),\ Lig(k,kp)=1} Q(s,r,r,kp,k) \quad - \\ \sum_{kp \,|\, kp \in \{k3\},\ Lig(k,kp)=1} \ \sum_{i \,|\, i \in \{r\}} Q(s,i,r,kp,k) \end{array}\right] \quad \forall r,k,s \,\middle|\, \begin{array}{l} k \in \{k1\}, \\ L0ik(r,k)=1 \end{array} \tag{15}$$

Equation (16) assesses the final quantity of material s stored in *k1* located at *r.*

$$Xf(s,r,k) = \left[\begin{array}{l} X(s,r,k) \quad + \\ \sum_{kp \,|\, kp \in \{k2\},\ oks(kp,s),\ Lig(kp,k)=1} Q(s,r,r,k,kp) \quad + \\ \sum_{kp \,|\, kp \in \{k3\},\ Lig(k,kp)=1} \ \sum_{i \,|\, i \in \{r\}} Q(s,r,i,k,kp) \end{array}\right] \quad \forall r,k,s \,\middle|\, \begin{array}{l} k \in \{k1\}, \\ L0ik(r,k)=1 \end{array} \tag{16}$$

The equation (17) defines the total storage used for process *k* at *r*.

$$QTstor(k,r) = \sum_{s} Xf(s,r,k) \quad \forall k,r \,\middle|\, \begin{array}{l} k \in \{k1\} \\ Capmax(r,k)>0 \end{array} \tag{17}$$

[7] This intermediate variable is created to assess the *k1* recovery and sorting materials after discounting the *k2* and *k3* input materials.

Equation (18) assesses the total quantity of materials and products *s* available

$$QsTo(s) = \sum_{k\ |k\in\{k1\}} \ \sum_{r\ |L0ik(r,k)=1} Xf(s,r,k) \qquad \forall s \tag{18}$$

Imposing that production meets the demand

Equation (19) enforces the production of *s* to satisfy the demand

$$QsTo(s) \geq Dem(s) \qquad \forall s \,|\, Dem(s) > 0 \tag{19}$$

Assessing the installation costs

Equation (20) computes the cost for the newly installed processing units

$$CIik(i,k) = (1 - E0ik(i,k)) \cdot (Inv(i,k) \cdot Yrsc(k,i)) \qquad \forall i,k \ \ |\ L0ik(i,k) = 1 \tag{20}$$

Equation (21) computes the investment total cost[8]

$$CInv = \sum_{i} \ \sum_{k\ |L0ik(i,k)=1} CIik(i,k) \tag{21}$$

Evaluating the transportation costs

Equation (22) computes the transport loading cost at *k0* and its transport to *k1*

$$Ck0(i) = \sum_{k\ |k\ \in\{k1\}} \ \sum_{kp\ |kp\ \in\{k0\}} \ \sum_{s\ |iks(k,s)} \ \sum_{j\ |j\ \in\{r\}} CLp(i,s) \cdot Q(s,j,i,k,kp) \qquad \forall i \tag{22}$$

Equation (23) computes the cost of transporting all materials *s* from *k0* at *i* to *k1* at *j*

$$Ck01(i,j) = \sum_{k\ |k\in\{k0\}} \ \sum_{kp\ |kp\in\{k1\}} \ \sum_{s\ |s\in iks(kp,s)} Lig(k,kp) \cdot Cps(i,s,j) \cdot Q(s,j,i,kp,k) \qquad \forall i,j \,|_{j\in\{r\}} \tag{23}$$

Equation (24) computes the total cost of transporting all materials *s* in transit on the route *I, j*

$$Ck1n(i,j) = \begin{bmatrix} \sum_{k\ |k\in\{k1\}} \sum_{kp\ |kp\in\{k2\}} \sum_{s\ |s\in iks(kp,s)} Cps(i,s,j) \cdot Lig(k,kp) \cdot Q(s,j,i,kp,k) \ + \\ \sum_{k\ |k\in\{k1\}} \sum_{kp\ |kp\in\{k2\}} \sum_{s\ |s\in oks(kp,s)} Cps(j,s,i) \cdot Lig(kp,k) \cdot Q(s,i,j,k,kp) \ + \\ \sum_{k\ |k\in\{k1\}} \sum_{kp\ |kp\in\{k3\}} \sum_{s\ |s\in iks(kp,s)} Cps(i,s,j) \cdot Lig(k,kp) \cdot Q(s,j,i,kp,k) \ + \\ \sum_{k\ |k\in\{k1\}} \sum_{kp\ |kp\in\{k3\}} \sum_{s\ |s\in oks(kp,s)} Cps(j,s,i) \cdot Lig(kp,k) \cdot Q(s,i,j,k,kp) \end{bmatrix} \qquad \forall i,j \,\Big|_{\substack{i\in\{r\} \\ j\in\{r\}}} \tag{24}$$

[8] Not accounted here the fiscal, operative and financial contributions for the investment.

Equation (25) computes the total cost of transportation

$$CostTr = \sum_{i} Ck0(i) + \sum_{i}\sum_{j} Ck01(i,j) + \sum_{i}\sum_{j} Ck1n(i,j) \tag{25}$$

Estimate of the processing costs

Equation (26) computes the processing cost for the material *s* at location *r* by process *k1*

$$CPk(r,k,s) = \sum_{i} \sum_{kp \mid kp\in\{k0\}} Lig(kp,k)\cdot Cop(r,k)\cdot Q(s,r,i,k,kp) \quad \forall r,k,s \,\Big|\, \begin{matrix} k\in\{k1\}, \\ s\in\{iks(k,s)\}\end{matrix} \tag{26}$$

Equation (27) computes the cost for processing the material *s* at location *r* by process *k2 and k3*

$$CPk(r,k,s) = \sum_{i \mid i\in\{r\}} \sum_{kp \mid kp\in\{k1\}} Lig(kp,k)\cdot Cop(r,k)\cdot Q(s,r,i,k,kp) \qquad \forall r,k,s \,\Big|\, \begin{matrix} k\in\{krf\}, \\ s\in iks(k,s)\end{matrix} \tag{27}$$

Equation (28) computes the total processing cost

$$CostPr = \sum_{r}\sum_{k}\sum_{s \mid s\in iks(k,s)} CPk(r,k,s) \tag{28}$$

Estimate of the reusable materials' voucher costs

Equation (29) computes the total cost of the reusable materials vouchers

$$CostVouch(s) = Voucher(s)\cdot \sum_{r \mid L0ik(r,k)>0} \sum_{k \mid k\in\{k1\}} Q(s,r,r,k,k) \quad \forall s \mid Voucher(s)>0 \tag{29}$$

Equation (29) computes the total cost of the reusable materials vouchers

$$TCVoucher = \sum_{s \mid Voucher(s)>0} CostVouch(s) \tag{30}$$

Estimate of the value of the products

Equation (31) assesses the *k1*, *k2* and *k3* product's value

$$Valuesk(s,k) = Vals(s)\cdot \sum_{r \mid L0ik(r,k)=1} Xf(s,r,k) \quad \forall k,s \,\Big|\, \begin{matrix} k\in\{k0\} \\ Vals(s)>0\end{matrix} \tag{31}$$

Equation (32) assesses the *k1*, *k2* and *k3* product's value

$$Valuesk(s) = \sum_{k \mid k\in\{k1\}} Valuesk(s,k) \qquad \forall s \,\Big|\, \begin{matrix} s\in\{k1Sell\} \\ Vals(s)>0\end{matrix} \tag{32}$$

Equation (33) gives the total value for all the network products

$$Value = \sum_{k \;|k \in \{kpr\}} \sum_{s \;|Vals(s)} Valuesk(s,k) \tag{33}$$

Objective function

Equation (34) computes the total cost

$$Cost = CInv + CostTr + CostPr \tag{34}$$

Equation (35) computes the total profit

$$Profit = Value - Cost - \sum_{s\{Vals(s)>0} CostVouch(s) \tag{35}$$