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(De)construct for Circular Economy

(Des)construir para a Economia Circular

WP 5 - Model

5.3 – Mathematical model of an inter-municipal CDW processing network

Final report

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Iceland Liechtenstein Norway grants

Authors:

Joaquim Duque – UEREE

LNEG – National Laboratory of Energy and Geology (Portugal)

Operador do Programa: REPÚBLICA PORTUGUESA AMBIENTE E AÇÃO CLIMÁTICA

Promotor: cimbal

Parceiros:

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List of acronyms used

AML - Lisbon Metropolitan Area.

CDW - Construction and demolition wastes

HQ - High-quality recycling facilities.

LQ - Low-quality recycling facilities.

RSS - Recovering sorting and storage facility

SR - Sorting and Recycling

1. Introduction

The main objective of the project **(De)construct for Circular Economy [in Portuguese: (Des)construir para a Economia Circular]** is to promote a regional strategy for the reuse of building products and components as well as the recycling of construction and demolition waste (CDW), thus reducing the environmental impact of the construction sector and promoting its circularity.

Task WP5 – Mathematical model of an inter-municipal CDW processing network:

The model for task 5.3 was developed under my coordination in an IST master thesis [1].

Given the scarcity/unavailability of regional data, this report presents only the mathematical model developed.

The report chapters are structured as follows:

- Introduction (present chapter);
- Objectives (chapter 2);
- Methodological approach (chapter 3);
- Model description (chapter 4);
- Conclusions (chapter xx);
- References (chapter xx)
- Mathematical Model (Annex I)

(This is the final version of the report.)

2. Objectives

The objective of this task is to develop a mathematical model to help the strategic, tactical and operational decisions for the implementation of an inter-municipal CDW recycling network of the materials locally produced by deconstruction (selective demolition) processes and the CDWs stored at a recovering, sorting and storage facility (see the report for tasks 5.1 and 5.2), to increase the circular economy. Depending on the conditions imposed, the model gives the optimal network design defining the optimal location to install the processing centres eventually required, and their respective capacities, and shows the optimized material flow in the network. The inclusion of the materials flow for landfilling is also modelled, allowing to impose of a minimal percentage of CDWs' recycling for the optimized solutions obtained.

3. Methodological approach

The development of the models uses a representation where the materials are univocally represented by states that flow in the network to be processed by the task performed in the network processing units/facilities. This representation is usually known in the specialized literature as a state task network, STN for short.

To ensure the model's generality, its implementation used a model core, constituted by the mathematical equations, and a parametrization warranted by external files. That file defines the model sets and parameters to be used for the particular scenario to be optimized. If several scenarios are to be studied a second file may be used to define the parameters varying with the scenario leaving the first one to deal with the immutable parameters. The latter are usually the ones defining the case study, namely the routes and facilities layout, process recipes, costs, values, etc...

The present model developed by Braga 2022 [1] expands the work from Andrade, 2015 [2] with a time discretization. The possible types of the network's centers are *a)* the sorting centers, *b)* the low-quality (LQ) recycling centers, *c)* the high-quality (HQ) recycling centers, and *d)* the landfilling facilities. The sorting facilities (K1) even if existing independently may gain a distinct identification (k2) if incorporated in a LQ recycling installation (K2).

To verify the model's suitability, executability, and general behaviour, an inter-municipal case study, not presented here, is used with two types of construction and demolition CDWs from all the parishes in the Lisbon Metropolitan Area (AML), Braga 2022 [1].

The model allows defining the existence of pre-installed centres/facilities and the installation of new ones according to the recycling demand level, their maximal capacity, the interconnections allowed, the routes and the transports used among others. The model optimization defines the optimal network design while assessing the costs for installation, operation, and transportation to optimise an objective function (*e.g.* minimize the total network cost).

4. Model description

The mathematical model uses a set of locations where the CDWs are produced/stored and where the diverse facilities are installed. The geographic level for these locations may be considered at a parish, municipal, or regional level.

The three levels of classification for the actual recycling centers, presented in figure 1, are according to the Symonds Group Ltd et al. 1999 [3], as cited by Correia, 2013 [2],

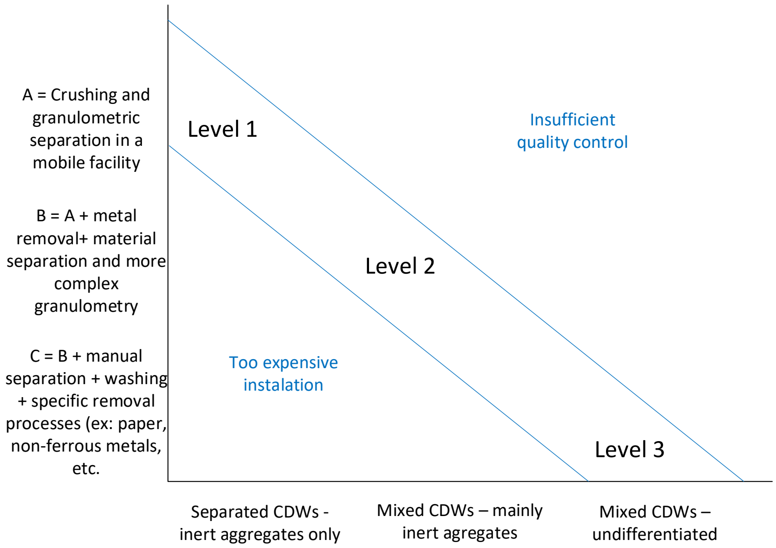

Figure 1 - Relating recycling facilities' level to their technical processes and type of CDWs processed.

The types of stationary centers considered in the model follow the simplification used in [3] by Andrade, 2015, from the definition of the diverse units/processes that constitute an ideal recycling facility presented in [4] by Coelho *et a*l, 2013.

Andrade realized that there are, in addition to the existing low-quality (LQ) recycling centers, other types that only carry out sorting work, designated as sorting centers. In addition, it was possible to perceive that there are no facilities in the Lisbon Metropolitan Area (AML)[1] that carry out high-quality (HQ) recycling. These HQ centers allow their recycled products to be used in a way similar to the products from which they were processed. Furthermore, he noted that they can be implemented in the existing centers, since they only need the space to allocate the new machines. It is noted here that the mobile type of recycling, using a crusher, is already considered as part of the units installed in the recovery, sorting, and storage (RSS) centers described in the report of joint tasks 5.1 and 5.2. In his work,

Thus the model facilities are the facilities for sorting, LQ recycling, HQ recycling, and landfilling that may landfill both the CDWs and the residual materials from all the processes, shown in figure 2.

[1] The model development used the AML region due to data availability, and the IST master students' manpower to help develop this field of work.

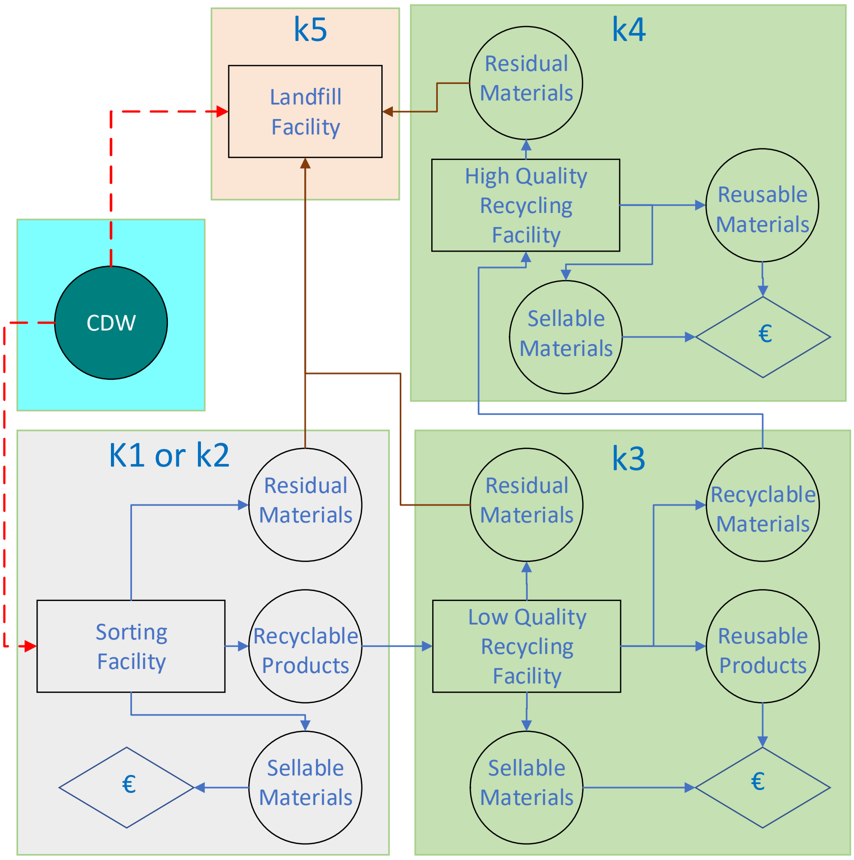

Figure 2 - Flow diagram of the CDWs and materials in a recycling network.

Figure 2 presents the flow diagram of the CDWs recycling network where the recycling processes are an alternative to landfilling as is established by the dotted red lines flows.

The model includes an input scalar, *Xrec*, allowing the imposition of a minimal percentage of CDW recycling. It is also assumed that all sellable materials that do not exceed the demand and unused reusable products are sold.

5. Model's testing

The model development expands the work from Andrade, 2015 [3] with a time discretization.

The present model, with time discretization, was developed by Braga 2022 [1] with data from AML, using the 211 parishes as nodes for the 12 months of the year 2015. That time discretization uses seasonal construction and demolition rates to obtain the monthly data from the annual values. It was tested in his master's thesis work and thoroughly reviewed by the thesis jury. The test results may be consulted in the referred thesis at the IST master thesis database.

6. Deliverables

Several attempts to get data from the municipal partners either at the teleconferences organized by CIMBAL or with direct contacts by email with CIMBAL, asking to reinforce next to the municipalities the need of obtaining and sending the needed data, lead to no result. Given the absence of data, the adaptation of the model to the inter-municipal region under analysis in the present project was not possible. However, the model's suitability, executability, and general behaviour were tested using an inter-municipal case study, not presented here, with two types of construction and demolition CDWs from all the parishes in the Lisbon Metropolitan Area (AML), Braga 2022 [1].

7. Conclusions

The mathematical model presented here is general since it was conceived to allow external parametrization of the core model by external files. One of these files may define the parameters configuring a given case study while in another file, or included in the previous one, the case study's scenarios may then be defined. The file(s) are then included in the general core model via the appropriate command.

The model allows defining the existence of pre-installed centres/facilities and the installation of new ones according to the recycling demand level, their maximal capacity, the interconnections allowed, the routes and the transports used among others.

The model optimization defines the optimal network design while assessing the costs for installation, operation, and transportation to optimise an objective function (e.g. minimize the total network cost).

The model is suitable for sensitive analysis studies of pertinent parameters such as landfilling costs or recycled quantities.

The time horizon of the model may easily be extended to help in the strategic decision process.

The main limitations arise from the computational time required to solve very large case studies, case studies larger than the one for the 211 nodes (parishes) of the AML with 12 discrete time intervals (months) over a time horizon of one year.

8. References

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[4] Coelho, A., & de Brito, J. (2013). Environmental analysis of a construction and demolition waste recycling plant in Portugal – Part I: location, materials, technology and economic analysis. Journal of Cleaner Production, 39, 338-352. DOI: https://doi.org/10.1016/j.jclepro.2012.08.024.

Annex I - Mathematical Model

The core of the mathematical model presented below describes a recycling network for CDW recyclable materials.

I.1 - Parametrization

Next are presented the sets, parameters, tables, and scalars of the model. They may be writen in an external file to be inserted in the model's core, presented in chapter I.2, to parametrize it.

I.1.1 - Model's sets

Note: the cardinals used in t the set elements presented next are the ones used for the AML case study.

Table 1 - Set s identifying all states

Set	Explanatory text	Set elements
i	- The network's locations, a set of *n* elements	$\{i_1,...,i_{211}\}$
s	- Material states	$\{s_1,...,s_{12},mr_1,mr_2\}$
k	- Processes	$\{k_0,...,k_5\}$
j	- Defines a set *j* identical to the set *i* by using	$alias(i,j)$
sp	- Defines a set *sp* identical to the set *s* by using	$alias(s,sp)$
kp	- Defines a set *kp* identical to the set *k* by using	$alias(k,kp)$
t	- Monthly periods	$\{t_1,...,t_{12}\}$
kc	- Available processe capacity's levels	$\{kc_1,...,kc_6\}$

Table 2 – Sub-sets to define instantiation and execution conditions in the model's equations

SubSet	Explanatory text	Set elements
$cdw(s)$	- A subset of CDWs	$\{s_1, s_2\}$
$mr(s)$	- A subset of residual materials for landfilling	$\{mr_1, mr_2\}$
$pi(s)$	- A subset of intermediate products	$\{s_3, s_4, s_{10}\}$
$pv(s)$	- A subset of reusable products	$\{s_5, \ldots, s_{11}\}$
$nv(s)$	- A subset of non-sellable products	$\{s_3, s_4, mr_1, mr_2\}$
$sl(s)$	- A products with unlimited selling quantity	$\{s_5, .., s_8\}$
$ka(k)$	- Sorting and Recycling Processes	$\{k_1, \ldots, k_4\}$
$kd(k)$	- All processes excluding the "dummy" $k0$	$\{k_1, \ldots, k_5\}$
$kf(k)$	- Sorting and landfilling processes	$\{k_1, k_2, k_5\}$
$kg(k)$	- "Dummy" and landfilling processes	$\{k_0, k_5\}$

Table 3 – Sub-sets to define instantiation and execution conditions in the model's equations (cont.)

SubSet (cont.)	Explanatory Text	Processes	States
$iks(k,s)$	The input material states *s* of processe *k*	$\{k_0, k_1, k_2\}$	$\{CDW\}$
		$\{k_3\}$	$\{s_3, s_4\}$
		$\{k_4\}$	$\{s_{10}\}$
		$\{k_5\}$	$\{s_{10}\}$
$oks(k,s)$	The output material states *s* of processe *k*	$\{k_0\}$	$\{CDW\}$
		$\{k_1, k_2\}$	$\{s_3, \ldots, s_8, mr_1, mr_2\}$
		$\{k_3\}$	$\{s_5, \ldots, s_{10}, mr_2\}$
		$\{k_4\}$	$\{s_5, \ldots, s_8, s_{11}, mr_2\}$
		$\{k_5\}$	$\{s_{12}\}$

I.1.2 - Parameters

Table 4 – Parameters used to define instantiation and execution conditions in the model's equations

Name	Explanatory text	Set elements
$cda(s)$	- Landfilling costs [€/ton]	$\{s_1, s_2, mr_1, mr_2\}$
$e(s)$	- Sell value [€/ton]	$\{s_5, \ldots, s_{11}\}$
$u(s,t)$	- Global sellable materials demand [ton/month]	$\forall s,t \; \lvert s \in \{s_5, \ldots, s_{11}\}$
$Percent(t)$	- Percentual CDW production per period *t*	$\{t_1, \ldots, t_{12}\}$
$ht(i,s,t)$	- CDW production per period *t* of CDW s at *i*	$\forall i,s,t \; \lvert s \in \{s_1, s_2\}$
$h(i,s)$	- Quantity of CDW of type *s* produced at *i* [ton/year]	$\forall i,s \; \lvert s \in \{s_1, s_2\}$
$v(k,s,sp)$	- Proportion of *sp* produced from *s* by a center of type *k*	$s \in iks(k,s),\; sp \in oks(k,s)$
$yp(i,k)$	- Pré-existing processes of type *k* at location *i*	$\forall i,k$
$cp(k,s)$	- Processing cost for material *s* at *k* [€/ton]	$k \in \{k_1, k_4\}$, $s \in \{s_1, \ldots, s_4, s_{10}\}$
$kup(i,k)$	- Existing capacity for process of type *k* at *i* [ton/year]	$\forall i,k$
$kkc(kc,k)$	- Max. capacity *kc* to install new processes *k* [ton/month]	$\forall kc,k \; \lvert k \in \{k_1, .., k_4\}$
$ikc(k,kc)$	- Install cost for a process *k* with capacity *kc* [€]	$\forall k,kc \; \lvert k \in \{k_1, .., k_4\}$
$Lig(k,kp)$	- Connections allowed processes from *k* to *kp*	$\forall k,kc \; \lvert k \in \{k_0, .., k_4\}$

I.1.3 - Scalar

Table 5 – A scalar to inpose the CDW recycling fraction

Name	Explanatory text	Value
$Xrec$	- Factor imposing a minimum value for CDW recycling	0.7

I.2 – Model's core

The model core is constituted by the definition of the variables used and the equations/inequations, definition and writing.

I.2.1 – Variables used

Table 6 – Binary variables used to define processes' existence and the capacity level used

Binary	*Explanatory Text*
$Y(k,i,t)$	- A binary variable[2] defining if at time interval *t* the processing units *k* is installed at *i*
$YC(k,kc,i,t)$	- A binary variable[3] defining that unit *k* installs with the capacity level, *kc*, in *i* at *t*

Table 7 – Real variables used

Real	*Explanatory Text*
$Cost$	- Network total cost [€]
$ProcCost(t)$	- Total cost of the processes [€/month]
$LandfillCost(t)$	- Landfill total cost [€/month]
$Proc1(t)$	- Process k_1 total cost [€/month]
$Proc2(t)$	- Process k_2 total cost [€/month]
$Proc3(t)$	- Process k_3 total cost [€/month]
$Proc4(t)$	- Process k_4 total cost [€/month]

Table 8 – Non-negative variables used

Non-negative	*Explanatory Text*
$Qrec$	- Quantity of CDWs recycled [ton/year]
$Qland$	- Quantity of CDWs landfilled [ton/year]
$Kki(k,i,t)$	- Installed capacity[4] for *k* at *i* at interval *t* [ton/month]
$Iki(k,i,t)$	- Investment to install *k* at *i* at interval *t*
$X(s,i,k,t)$	- Quantity of materials *s* produced by *k* sold at location *i* at interval *t* [ton/month]
$Q(s,i,j,k,kp,t)$	- Quantity of materials *s* going from *k* at *i* to *kp* at *j* at interval *t* [ton/month]
$hsto(i,s,t)$	- Quantity of material of type *s* arriving to storage at *i* at interval *t*
$QA(i,s,t)$	- Quantity of material of type *s* stored at *i* at interval *t*

[2] Once installed the model forces its continuity over the intervals that follow.
[3] The same as above for the capacity level installed.
[4] Once installed the model forces its continuity over the intervals that follow

I.2.2 – Equations presentation

This section presents all the equations/inequations used along with a brief explanation of its function in the mathematical model.

Table 9 –Presentation and function of the equations used in the model

ID	*Explanatory Text*
Eq1	Objective function
Eq2(t)	Computes the net cost for all processes (including the selling of products)
Eq3(t)	Computes the cost for the CDW sent directly to landfilling
Eq4(t)	Cost of processing and transporting materials
Eq5t)	Cost of RM deposition and transport
Eq6(t)	Profit from the sale of materials
Eq7(t)	Cost of investment
Eq8(k,i,t)	Forces the permanency of the pre-existing processes in interval t1
Eq9(k,i,t)	Forces the permanency of the existing processes for $t \in \{t_2,...,t_{12}\}$
Eq10(k,i,t)	Disallows the opening of new landfills and new “dummy” K_0 processes
Eq11(k,i,t)	Allows only one of the possible capacities to the pre-existing sorting and recycling processes at interval t_1
Eq12 (k,kc,i,t)	Maintains the capacity *kc* previously chosen for *k*
Eq13(k,i,t)	Ensures the permanency of the open sorting and recycling processes
Eq14(k,i,t)	Assigns the capacity values to pre-existing sorting and recycling processes
Eq15(k,i,t)	Assigns the capacity values for the “dummy”, k_0, and landfill, k_5, processes
Eq16(k,i,t)	Assigns the capacity values to newly installed sorting and recycling processes
Eq17(k,i,t)	Computes the investment cost of additional sorting and recycling processes at t_1
Eq18(k,i,t)	Computes the investment cost of newly installed sorting and recycling processes at $t>t_1$
Eq19(k,i,t)	Forces the investment cost of the pre-existing centers to be zero
Eq20(s,t)	limits the sellable quantities production by the demand
Eq21(kp,j,t)	Limits the quantity of the material arriving at *kp* center to its capacity
Eq22	Computes the annual quantity of CDWs recycled
Eq23	Computes the annual quantity of CDWs landfilled
Eq24	Imposes a percentage of recycling
Eq25	Computes the percentage of landfilling
Eq26(i,s,t)	The stored quantity of CDW *s* at *i* at *t* allows the leaving quantity
Eq27(j,kp,sp,t)	Performs a mass balance for *kp* at *j*, where the X(sp,j,kp,t) value is computed
Eq28(i,s)	Computes the quantity of CDW, *s*, arriving to store at *i*, *at interval* t_1
Eq29(i,s,t)	Computes the quantity of CDW, *s*, arriving to store at *i* at interval $t \in \{t_2,...,t_{12}\}$
Eq30(i,s,t)	Computes the final quantity of CDW of type*s* stored at *i* at interval *t*

I.2.3 – Equations definition

This section presents the writing of all the equations/inequations in a mathematical format preceded by a brief explantion of their function.

Equation (1) computes the net total cost, used as the objective function.

$$Cost = \sum_t \left(ProcCost(t) + LandfillCost(t) \right) \tag{1}$$

Equation (2) computes the net cost for all processes (including the selling of products).

$$ProcCost(t) = Proc1(t) + Proc2(t) - Proc3(t) + Proc4(t) \qquad \forall t \tag{2}$$

Equation (3) computes the cost for the CDW sent directly to landfilling.

$$LandfillCost(t) = \sum_i \sum_j \sum_{s|s \in \{CDW(s)\}} \sum_{k|k \in \{k_0\}} \sum_{kp|kp \in \{k_5\}} \left(\left(cda(s) + b(i,j) \right) \cdot Q(s,i,j,k,kp,t) \right) \;\; \forall t \tag{3}$$

Equation (4) computes the cost of processing and transporting materials.

$$Proc1(t) = \sum_i \sum_j \sum_{kp\;|kp \in \{ka(kp)\}} \sum_{s\;|s \in \{iks(kp,s)\}} \sum_{k\;|k \in \{oks(k,s)\}} Lig(k,kp) \cdot \left(b(i,j) + cp(kp,s) \right) \cdot Q(s,i,j,k,kp,t) \;\; \forall t \tag{4}$$

Equation (5) computes the cost of the *RM* deposition and transport.

$$Proc2(t) = \sum_i \sum_j \sum_{s|s \in \{mr(s)\}} \sum_{k|k \in \{oks(k,s)\}} \sum_{kp|kp \in \{k5\}} \left(\left(cda(s) + b(i,j) \right) \cdot Q(s,i,j,k,kp,t) \right) \;\; \forall t \tag{5}$$

Equation (6) computes the profit from the sale of materials

$$Proc3(t) = \sum_{k|k \in \{ka(k)\}} \sum_i \sum_{s\;|s \in \{pv(s) \cap oks(k,s)\}} \left(X(s,i,k,t) \cdot e(s) \right) \;\; \forall t \tag{6}$$

Equation (7) computes the cost of investment

$$Proc4(t) = \sum_{k|k \in \{ka(k)\}} \sum_i IKI(k,i,t) \;\forall t \tag{7}$$

Equation (8) forces the permanency of the processes pre-existing at interval *t1*

$$Y(k,i,t) = yp(i,k) \quad \forall k,i,t \;\; |k \in \{ka(k)\},\, t \in \{t1\} \tag{8}$$

Equation (9) forces the permanency of the processes existing in the previous time interval.

$$Y(k,i,t) \geq Y(k,i,t-1) \quad \forall k,i,t \;\; |k \in \{ka(k)\},\, t \in \{t_2,\ldots,t_{12}\} \tag{9}$$

Equation (10) disallows the opening of new landfills and new “dummy” k_0 processes.

$$Y(k,i,t) = yp(i,k) \quad \forall k,i,t \;\; |k \in \{kg(k)\} \tag{10}$$

Equation (11) atributes one of the possible capacities to the pre-existing *SR* processes at $t \in \{t1\}$.

$$\sum_{kc} YC(k,kc,i,t) = Y(k,i,t) \quad \forall k,i,t \; \left| k \in \{ka(k)\},\ yp(i,k)=0,\ t \in \{t1\} \right. \tag{11}$$

Equation (12) maintains the capacity *kc* previously chosen for *k*.

$$YC(k,kc,i,t) \geq YC(k,kc,i,t-1) \quad \forall k,kc,i,t \; \left| \begin{array}{l} k \in \{ka(k)\}, \\ yp(i,k)=0, \\ t \in \{t_2,\ldots,t_{12}\} \end{array} \right. \tag{12}$$

Equation (13) ensures the permanence of the open sorting and recycling (SR) processes.

$$\sum_{kc} YC(k,kc,i,t) = Y(k,i,t) \quad \forall k,i,t \; \left| \begin{array}{l} k \in \{ka(k)\}, \\ yp(i,k)=0, \\ t \in \{t_2,\ldots,t_{12}\} \end{array} \right. \tag{13}$$

Equation (14) assigns the capacity values to pre-existing sorting and recycling processes.

$$KKI(k,i,t) = kup(i,k) \quad \forall k,i,t \quad \left| k \in \{ka(k)\},\ yp(i,k)=1 \right. \tag{14}$$

Equation (15) assigns the capacity values for the “dummy”, k_0, and landfill, k_5, processes.

$$KKI(k,i,t) = kup(i,k) \cdot yp(i,k) \quad \forall k,i,t \; \left| k \in \{kg(k)\} \right. \tag{15}$$

Equation (16) assigns the capacity values to newly installed sorting and recycling processes.

$$KKI(k,i,t) = \sum_{kc} kkc(kc,k) \cdot YC(k,kc,i,t) \quad \forall k,i,t \; \left| k \in \{ka(k)\},\ yp(i,k)=0 \right. \tag{16}$$

Equation computes the investment cost of additional sorting and recycling processes at t_1.

$$IKI(k,i,t) = \sum_{kc} ikc(k,kc) \cdot YC(k,kc,i,t) \quad \forall k,i,t \; \left| ka(k),\ yp(i,k)=0,\ t \in \{t_1\} \right. \tag{17}$$

Equation (18) computes the investment cost of newly installed *SR* processes at $t \in \{t_2,\ldots,t_{12}\}$.

$$IKI(k,i,t) = \sum_{kc} ikc(k,kc) \cdot \left(YC(k,kc,i,t) - YC(k,kc,i,t-1) \right) \quad \forall k,i,t \; \left| \begin{array}{l} k \in \{ka(k)\}, \\ yp(i,k)=0, \\ t \in \{t_2,\ldots,t_{12}\} \end{array} \right. \tag{18}$$

Equation (19) forces the investment cost of the pre-existing centers to be zero.

$$IKI(k,i,t) = 0 \quad \forall k,i,t \; \left| yp(i,k)=1 \right. \tag{19}$$

Equation (20) limits the sellable quantities by the demand.

$$\sum_{k | k \in \{ka(k)\}} \sum_{i} X(s,i,k,t) \leq u(s,t) \quad \forall s,t \tag{20}$$

Equation (21) limits the quantity of the material arriving at *kp* center to its capacity.

$$KKI(kp,j,t) \geq \sum_{s \,|s\in\{iks(kp,s)\}} \sum_{k \,|k\in\{oks(k,s)\}} \sum_{i} \left(Lig(k,kp)\cdot Q(s,i,j,k,kp,t) \right) \quad \forall kp,j,t \;\Big|_{kp\in\{kd(kp)\}} \tag{21}$$

Equation (22) computes the annual quantity of *CDWs* recycled.

$$Qrec = \sum_{s \,|s\in\{CDW(s)\}} \sum_{i} \sum_{j} \sum_{k \,|kp\in\{k_0\}} \sum_{kp \,|kp\in\{k_1,k_2\}} \sum_{t} Q(s,i,j,k,kp,t) \tag{22}$$

Equation (23) computes the annual quantity of *CDWs* landfilled.

$$Qland = \sum_{s \,|s\in\{CDW(s)\}} \sum_{i} \sum_{j} \sum_{k \,|k\in\{k_0\}} \sum_{kp \,|kp\in\{k_5\}} \sum_{t} Q(s,i,j,k,kp,t) \tag{23}$$

Equation (24) imposes a percentage of recycling by using the factor $Xrec$.

$$Qrec = Xrec \cdot \sum_{i} \sum_{s \,|s\in\{CDW(s)\}} h(i,s) \tag{24}$$

Equation (25) computes the correspondent percentage of landfilling.

$$Qland = (1 - Xrec)\cdot \sum_{i} \sum_{s \,|s\in\{CDW(s)\}} h(i,s) \tag{25}$$

Equation (26) guaranties that the stored quantity of *CDW*, *s*, at *i* at interval *t* meets the leaving quantity.

$$hdep(i,s,t) \geq \sum_{j} \sum_{k \,|k\in\{k_0\}} \sum_{kp \,|kp\in\{kf(kp)\}} Q(s,i,j,k,kp,t) \quad \forall i,s,t \;\Big|_{s\in\{CDW(s)\}} \tag{26}$$

Equation (27) performs a mass balance for *kp* at *j*, where the *X(sp,j,kp,t)* value is computed.

$$X(sp,j,kp,t) = \left[\begin{array}{l} \displaystyle\sum_{s \,|s\in\{iks(kp,s)\}} v(kp,s,sp)\cdot \left(\sum_{i} \sum_{k \,|k\in\{oks(k,s)\}} Lig(k,kp)\cdot Q(s,i,j,k,kp,t) \right) \\ \displaystyle - \sum_{i} \sum_{k \,|k\in\{iks(k,sp)\}} Q(sp,j,i,kp,k,t) \end{array} \right] \quad \forall sp,j,kp,t \;\Big|_{\substack{kp\in\{kd(kp)\},\\ sp\in\{oks(kp,sp)\}}} \tag{27}$$

Equation (28) computes the quantity of *CDW*, *s*, arriving to store at *i*, at t_1.

$$\sum_{t \,|t\in\{t_1\}} hsto(i,s,t) = \sum_{t \,|t\in\{t_1\}} ht(i,s,t) \quad \forall i,s \tag{28}$$

Equation computes the quantity of *CDW*, *s*, arriving to store at *i*, at interval $t \in \{t_2,\ldots,t_{12}\}$.

$$hsto(i,s,t) = ht(i,s,t) + hsto(i,s,t-1) - \sum_{j} \sum_{k \,|k\in\{k_0\}} \sum_{kp \,|kp\in\{kf(kp)\}} Q(s,i,j,k,kp,t-1) \quad \forall i,s,t \;\Big|_{t\in\{t_2,\ldots,t_{12}\}} \tag{29}$$

Equation (30) computes the final quantity of *CDW* of types stored at *i* at interval *t*.

$$QA(i,s,t) = hsto(i,s,t) - \sum_{kp \,|kp \in \{kf(kp)\}} \sum_{j} Q(s,i,j,'k0',kp,t) \quad \forall i,s,t \tag{30}$$